A. Lesson Context

BIG PICTURE of this UNIT:	• How & why do we build NEW knowledge in Mathematics? • What NEW IDEAS & NEW CONCEPTS can we now explore with specific references to QUADRATIC FUNCTIONS? • How can we extend our knowledge of FUNCTIONS, given our BASIC understanding of Functions?		
CONTEXT of this LESSON:	Where we've been In Lessons 2 & 3, you worked with quadratic graphs and equations using the form of $y = \boldsymbol{a}(x - \boldsymbol{h})^2 + \boldsymbol{k}$	Where we are HOW do we apply the vertex form of quadratic models in contextual problems	Where we are heading How do we extend our knowledge & skills of quadratic functions, given the new ideas & concepts we now know about functions.

B. Lesson Objectives

a. Apply the equation $y = \boldsymbol{a}(x - \boldsymbol{h})^2 + \boldsymbol{k}$ to geometrical contexts and to modeling contexts as well as data contexts (scatter plots)

C. Fast Five (Skills Review Focus)

Solve the equation $0 = -2(x + 2)^2 + 32$

Find the vertex of the parabola $y = x^2 + 6x - 7$

A parabola with the equation of $y = a(x - h)^2 - 2$ goes through the points (1,2) and (-5,-1). Find the values of *a* and *h*.

D. Modeling with Quadratic Functions (Example #1 – Geometric Context)

A quadratic function is defined by the equation $f(x) = x^2 - 4x - 5$.

(a) Determine the equation of the axes of symmetry.
(b) Determine the vertex of this parabola.
(c) Rewrite the equation in vertex form and state the optimal value of the quadratic function.
(d) Find the zeroes of the parabola.
(e) Sketch the parabola, labelling the vertex and the y-intercept.
(f) Solve $f(x) = -5$.
(g) Using your answer from Q(c), write the equation of the INVERSE RELATION of f(x).

E. <u>Modeling with Quadratic Functions (Example #2)</u>

Mr S's sister is a motorcycle instructor and runs a training school. Because she works for herself, she can charge any amount (as an hourly charge) that she wishes. She keeps track of her hourly fees and her profits and has prepared a DATA TABLE showing the relationship between her hourly wages and her profits.

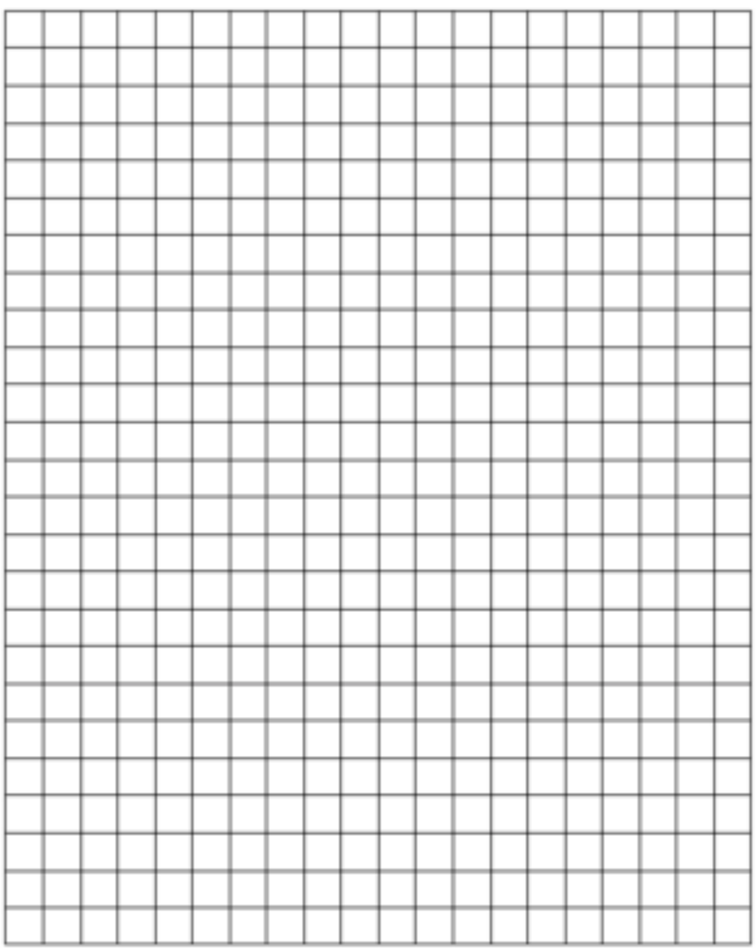
Here is her data set

Hourly fee	10	22	31	17	45	51	38
profit	1300	1750	1700	1600	950	400	1400

In this relation, the variables:

X represents the hourly fee my sister charges

Y represents the monthly profit she makes



(a) Graph the data points

(b) Draw the parabola that best fits the data set as well as you can.

(c) Determine the equation for this relation. Show your work.

(d) My sister would like to know what hourly fee OPTIMIZES her profits?

(e) From your equation in question (c), expand and rewrite on standard form

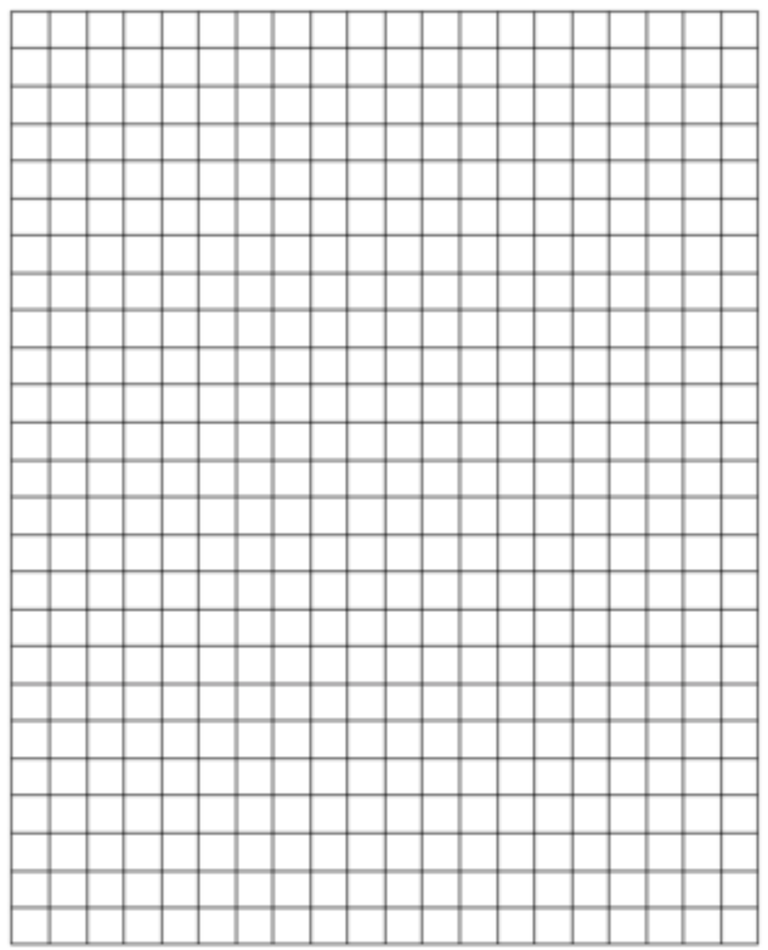
(f) Use your TI-84 to determine the equation of the regression curve that best fits your data set.

F. Modeling with Quadratic Functions (Example #3 – Modeling Businesses)

1. A chain of ice cream stores sells $840 of ice cream cones per day. Each ice cream cone costs $3.50. Market research shows the following trend in revenue as the price of an ice cream cone is reduced.

Price ($)	3.50	3.00	2.50	2.00	1.50	1.00	0.50
Revenue ($)	840	2520	3600	4080	3960	3240	1920

a) Create a scatter plot, and draw a quadratic curve of good fit.
b) Determine an equation in vertex form to model this relation.
c) Use your model to predict the revenue if the price of an ice cream cone is reduced to $2.25.
d) To maximize revenue, what should an ice cream cone cost?
e) Check the accuracy of your model using quadratic regression.



G. Modeling with Quadratic Functions (Examples #4 – Modeling Bridges & Rockets)

. The Lion's Gate Bridge in Vancouver, British Columbia, is a suspension bridge that spans a distance of 1516 m. Large cables are attached to the tops of the towers, 50 m above the road. The road is suspended from the large cables by many smaller vertical cables. The smallest vertical cable measures about 2 m. Use this information to determine a quadratic model for the large cables.

. A model rocket is launched from the ground. After 20 s, the rocket reaches a maximum height of 2000 m. It lands on the ground after 40 s. Explain how you could determine the equation of the relationship between the height of the rocket and time using two different strategies.

H. Modeling with Quadratic Functions (Examples #5 – Modeling Bridges & Rockets)

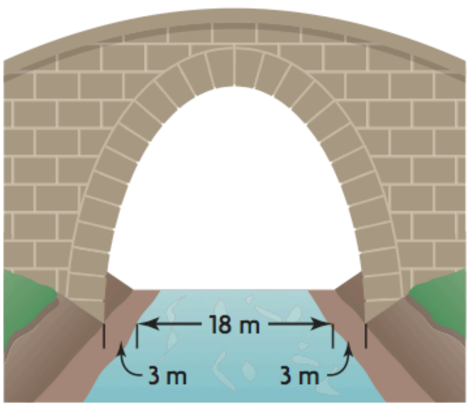
13. The average ticket price at a regular movie theatre (all ages) from 1995 to 1999 can be modelled by $C = 0.06t^2 - 0.27t + 5.36$, where C is the price in dollars and t is the number of years since 1995 ($t = 0$ for 1995, $t = 1$ for 1996, and so on).

a) When were ticket prices the lowest during this period?

b) What was the average ticket price in 1998?

c) What does the model predict the average ticket price will be in 2010?

d) Write the equation for the model in vertex form.



14. A bridge is going to be constructed over a river. The underside of the bridge will form a parabolic arch, as shown in the picture. The river is 18 m wide and the arch will be anchored on the ground, 3 m back from the riverbank on both sides. The maximum height of the arch must be between 22 m and 26 m above the surface of the river. Create two different equations to represent arches that satisfy these conditions. Then use graphing technology to graph your equations on the same grid.

I. Modeling with Quadratic Functions (Example #6 – Modeling Bridges)

The underside of a concrete railway underpass forms a parabolic arch. The arch is 24 m wide at the base and 8.0 high in the center. The upper surface of the underpass is 40 m wide. The concrete is 2 m thick at the center. Can a truck that is 5 m wide and 7.5 tall get through this underpass if the time is 3:30pm?

(a) Visualize the information by drawing a diagram wherein you label the relevant information.

(b) Recall that our general starting point would be a quadratic model and that the vertex form of the equation is $y = a(x - h)^2 + k$

(c) How can you use the quadratic equation to address the problem of the truck passing through the underpass?

J. Modeling with Quadratic Functions (Example #7)

The Next Cup coffee shop sells a special blend of coffee for $2.60 per mug. The shop sells about 200 mugs per day at this price. Customer surveys show that for every $0.05 decrease in price, the shop would sell 10 more mugs per day.

(HINT: start with numbers and a data table to see what may be going on)

(d) Determine the REVENUE that the coffee shop makes initially, given the price per mug and the amount of mugs sold.
(e) Since we are making changes in the pricing & revenues of the coffee shop, we need to decide upon an INDEPENDENT variable to use in modeling a change in the revenues ➔ so we need an R(x) equation …..
(f) Determine the MAXIMUM daily revenue from coffee sales and the price per mug in order to earn this revenue.
(g) Write an equation in both standard form and vertex form to model this problem. Then sketch the graph.