(A) Lesson Context

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I extend my knowledge of Quadratic Algebra?		
CONTEXT of this LESSON:	Where we've been In IM2, you studied quadratic functions from the perspective of graphs, algebra & modeling	Where we are Review features of quadratic graphs & quadratic algebra	Where we are heading How can I extend my algebra skills to analyze and model with quadratic functions?

(B) Lesson Objectives:

a. Review the key features of the graphs of Quadratic functions
b. Review the algebraic skills of involved in the analysis and applications of Quadratic Functions
c. Re-establish the connection between the key features of the graphs and the quadratic algebra skills
d. Incorporate the new function ideas of transformations and inverses

(C) Key Features of Parabolas

A ball is thrown from the rooftop of a building and the relationship between its height (in meters) and time of flight (in seconds) is modeled by the equation $h(t) = -5t^2 + 5t + 30$. Graph the function on your TI-84 and determine:

(i) Window Settings on TI-84

(ii) How tall is the building?

(iii) When will the ball hit the ground?

(iv) the domain and range (in context)

(v) When does the ball reach is highest point? How high?

(vi) Sketch the function

(vii) Explain how you could use algebraic skills to perform the SAME analysis

Use your TI-84 to graph $f(x) = -2x^2 + 6x + 10$ in an appropriate view window and then use the TI-84 to determine the following. Finally, sketch the parabola, labeling the key points/features.

(i) Window Settings on TI-84

(ii) Zeroes

(iii) Axis of Symmetry

(iv) Optimal point

(v) y-intercept

(vi) Sketch

Use your TI-84 to graph $f(x) = \frac{1}{2}(x-4)(x+10)$ in an appropriate view window and then use the TI-84 to determine the following. Finally, sketch the parabola, labeling the key points/features.

(i) Window Settings on TI-84

(ii) Zeroes

(iii) Axis of Symmetry

(iv) Optimal point

(v) y-intercept

(vi) Sketch

Use your TI-84 to graph $f(x) = 2(x+4)^2 - 18$ in an appropriate view window and then use the TI-84 to determine the following. Finally, sketch the parabola, labeling the key points/features.

(i) Window Settings on TI-84

(ii) Zeroes

(iii) Axis of Symmetry

(iv) Optimal point

(v) y-intercept

(vi) Sketch

(D) Practice with Quadratic Algebra

Given the quadratic function $f(x) = x^2 - 3x - 10$, answer the following using ALGEBRAIC methods (no TI-84s)

(i) Evaluate $f(-3)$

(ii) Factor the equation for $y = f(x)$ (rewrite in factored form)

(iii) HENCE, solve $0 = x^2 - 3x - 10$ (i.e. find the zeroes of y = f(x))

(iv) HENCE, determine the optimal point of $y = f(x)$

(v) Solve $f(x) = 18$ (i.e. solve $18 = x^2 - 3x - 10$)

Given the quadratic function $f(x) = 2(x-6)(x+10)$, answer the following using ALGEBRAIC methods (no TI-84s)

(i) Evaluate $f(-3)$

(ii) Solve $0 = 2(x-6)(x+10)$ (i.e. find the zeroes of y = f(x))

(ii) Expand the equation for $y = f(x)$ (rewrite in standard form)

(iv) HENCE, determine the optimal point of $y = f(x)$

(v) Solve $f(x) = -30$ (i.e. solve $-30 = 2(x-6)(x+10)$)

Given the quadratic function $f(x) = -4(x-2)^2 + 16$, answer the following using ALGEBRAIC methods (no TI-84s)

(i) Evaluate $f(-3)$

(ii) CONNECTIONS: Determine the transformations that were applied to the parent function of $f(x) = x^2$

(iii) HENCE, determine the optimal point of $y = f(x)$

(iv) CONNECTIONS: Solve $0 = -4(x-2)^2 + 16$ (i.e. find the zeroes of y = f(x) HINT: inversing the operations)

(v) CONNECTIONS: Write the equation for the inverse of $y = f(x)$

(vi) Expand the equation for $y = f(x)$ (rewrite in standard form)

(vii) Solve $f(x) = -20$ (i.e. solve $-20 = -4(x-2)^2 + 16$)

(E) Changing from Standard Form to Factored Form

Directions: USE A SEPARATE SHEET OF PAPER. Please factor the following expressions. If any of the following expressions cannot be factored, please indicate so by stating "prime".

1. x^2+5x+4
2. $x^2+12x+32$
3. $x^2+15x+50$
4. $a^2-5a-24$
5. $a^2+5a-24$
6. $r^2+2r-48$
7. $x^2+6x-72$
8. $d^2+2d+80$
9. x^2-6x+9
10. $m^2+15m+54$
11. $x^2-33x+32$
12. $x^2-12x+20$
13. b^2+b-72
14. $d^2-25d+156$
15. $b^2-10b+24$
16. $f^2-11f-26$

Directions: USE A SEPARATE SHEET OF PAPER. Please factor the following expressions. If any of the following expressions cannot be factored, please indicate so by stating "prime".

1. $6x^2-13x-5$
2. $3x^2+10x-25$
3. $10x^2+17x+3$
4. $6x^2-7x-3$
5. $12x^2-28x-5$
6. $3x^2-32x+45$
7. $14x^2-9x+1$
8. $12x^2-8x-15$
9. $11x^2+35x+6$

(F) Solving Quadratic Equations ➔ Application Problems

11. A model rocket is shot into the air and its path is approximated by $h = -5t^2 + 30t$, where h is the height of the rocket above the ground in metres and t is the elapsed time in seconds.
- **(a)** When will the rocket hit the ground?
- **(b)** What is the maximum height of the rocket?

12. A baseball is thrown from the top of a building and falls to the ground below. Its path is approximated by the relation $h = -5t^2 + 5t + 30$, where h is the height above ground in metres and t is the elapsed time in seconds.
- **(a)** How tall is the building?
- **(b)** When will the ball hit the ground?
- **(c)** When does the ball reach its maximum height?
- **(d)** How high above the building is the ball at its maximum height?

13. **Application:** A small company that manufactures snowboards uses the relation $P = 162x - 81x^2$ to model its profit. In the model, x represents the number of snowboards in thousands, and P represents the profit in thousands of dollars.
- **(a)** What is the maximum profit the company can earn?
- **(b)** How many snowboards must it produce to earn this profit?
- **(c)** The company breaks even when there is neither a profit nor a loss. What are the break-even points for the company?

14. A computer software company models the profit on its latest game using the relation $P = -2x^2 + 28x - 90$, where x is the number of games it produces in hundred thousands and P is the profit in millions of dollars.
- **(a)** What is the maximum profit the company can earn?
- **(b)** How many games must it produce to earn this profit?
- **(c)** What are the break-even points for the company?

(G)Practice – Graphing & Word Problem Context

Apply to Problems ➔ Mr. S. can sell 500 apples per week when he charges 50 cents per apple. Through market research, his wife (being smarter than Mr. S of course) knows that for every price increase of 2 cents per apple, he will sell 10 less apples.

i. Determine an equation that can you used to model Mr. S.'s expected revenues.

ii. What price should he charge to maximize his revenues?

iii. What is his maximum revenue?

iv. How many price increments are required such that his business has NO revenue?

Apply to Problems ➔ The profits of a company in its first 13 months of operations are modelled by the quadratic function $P(m) = -0.25m^2 + 3m - 5$ where m is the number of months (and m = 1 represents January) and P(m) is measured in billions of pesos. (CALC INACTIVE)

a. Determine when the company "breaks even".

b. Determine in which month the company maximizes its profits.

c. What are the company's maximum profits?

d. Solve and interpret P(m) < 0 given that the domain is $D: \{m \in Z \mid 0 \le m \le 13\}$

e. For what values of m are the profits DECREASING? Explain how you determined your answer.

f. Solve P(m) = -12 and interpret

18. **Thinking, Inquiry, Problem Solving:** Soundz Inc. makes CD players. Last year, accountants modelled the company's profit by $P = -5x^2 + 60x - 135$. Over the course of the year, in an effort to become more efficient, Soundz Inc. restructured its operation, eliminating some employees and reducing costs. This year, accountants are using $P = -7x^2 + 70x - 63$ to project the company's profit. In both models, P is the profit in hundreds of thousands of dollars and x is the number of CD players made, in hundreds of thousands. Was Soundz Inc.'s restructuring effective? Justify your answer.