A. Lesson Context

BIG PICTURE of this UNIT:	• How & why do we build NEW knowledge in Mathematics? • What do we mean by the term "function" • How do we work with the concept of "functions" given our prior experience with linear, quadratic and exponential relations from IM2? • How do we apply the function concept to real world models?		
CONTEXT of this LESSON:	Where we've been In Lessons 1&2, you learned about basic features of functions, including domain, range, increase, decrease, optimal points	Where we are NOW we will focus on addressing the idea of Inverses of Functions → what ARE inverses of functions & how do we work with inverses of fcns	Where we are heading What are functions, how do we work with them & how do we introduce new concepts stemming from "functions"?

B. Lesson Objectives

a. Introduce the concept of inverse functions through explorations
b. Introduce the inverse function notations
c. Apply the concept of inverse linear & quadratic & exponential functions through a variety of representations & contexts

C. FAST FIVE: Skills Review

(a) Solve for x if $-3 = 2x + 5$

(a) Solve for x if $y = 2x + 5$

(b) Solve for x if $3x - 2 = 5$

(b) Solve for x if $3x - 2y = 5$

(c) Solve for x if $8 - 5 = 3(x - 2)$

(c) Solve for x if $y - 5 = 3(x - 2)$

(d) Solve for x if $3(x + 2)^2 + 5 = 32$

(d) Solve for x if $3(x + 2)^2 + 5 = y$

D. Exploration Part #1:

Bob bumps his head and starts plotting all his data points in "reverse" order. For example, when he tries to plot (3,2), he plots (2,3) instead. A problem in his textbook tells him to graph the line y = 3x + 2

(a) List some of the points that would be on the line y = 3x + 2. Draw the CORRECT line.
(b) List some points that Bob would use in his line. Draw Bob's line.
(c) What is the slope of Bob's line? What is the equation of Bob's line?

E. Exploration #2: Number Patterns ➔ Going forward and Going back!!

For the following data tables or number sets, write an algebraic equation of the pattern, using function notation

x	f(x)
1	3
2	5
3	7
4	9

As equation: f(x) =

x	f(x)
2	23
3	19
4	15
5	11

As equation: f(x) =

x	f(x)
2	4
4	9
6	14
8	19

As equation: f(x) =

x	f(x)
1	9
2	16
3	25
4	36

As equation: f(x) =

x	f(x)
3	10
4	17
5	26
6	37

As equation: f(x) =

x	f(x)
6	7
9	13
12	19
15	25

As equation: f(x) =

x	f(x)
2	11
3	18
4	27
5	38
6	51

As equation: f(x) =

x	f(x)
2	16
3	4
4	1
5	?
6	?

As equation: f(x) =

x	f(x)
17	5
20	10
23	20
26	?
29	?

As equation: f(x) =

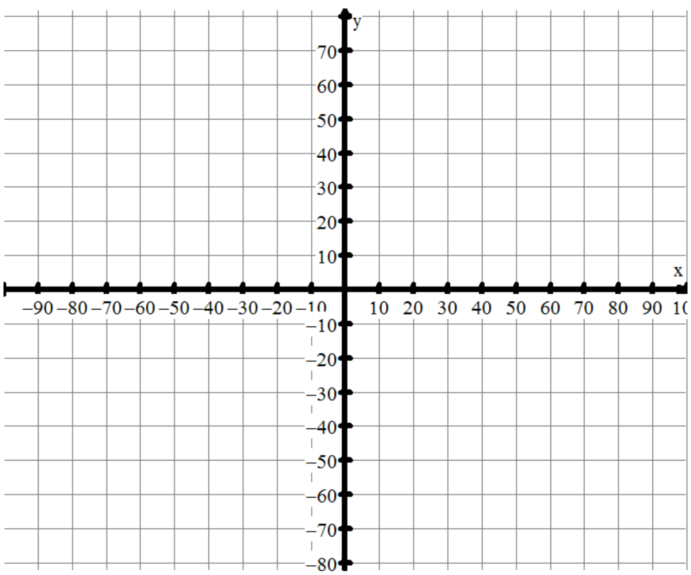
F. <u>Contextual Exploration – PART 1</u>

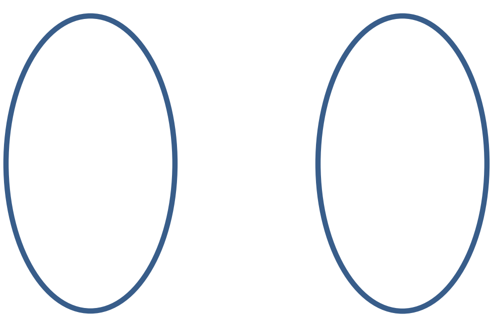
For American tourists visiting Canada, temperature data might seem a bit unusual. So a simplified "rule of thumb" for converting a temperature in degrees Celsius into degrees Fahrenheit is to double the Celsius temperature and then add 30.

(a) Copy and complete the table using the visitor's rule.

°C	-10	0	10	20	30	40
°F						

(b) Graph the relation.





(c) What is the independent variable?

(d) What is the dependent variable?

(e) Does our "temperature conversion rule" define a function? Explain.

(f) Let f represent the rule. What ordered pair, (0,????), belongs to f ?

(g) Let x represent the temperature in degrees Celsius. Write the equation for this rule in function notation.

(h) In the table, f (10) = 50, which corresponds to a point on the graph of y = f (x). What is the x-coordinate of this point? What is its y -coordinate?

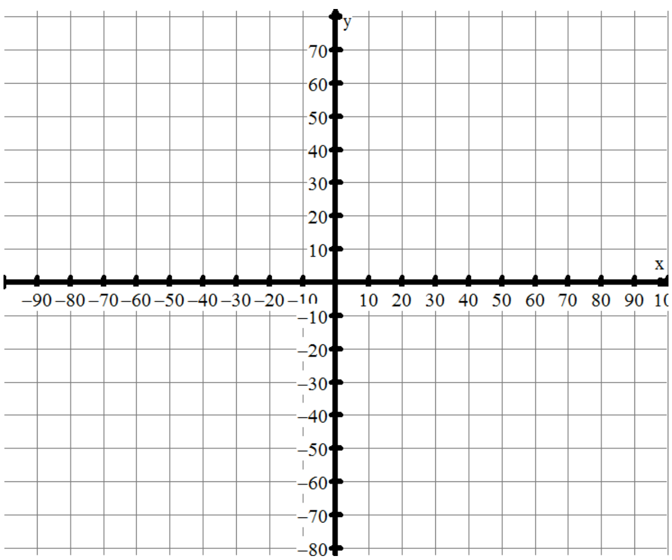
G. <u>Opening Exercise – PART 2</u>

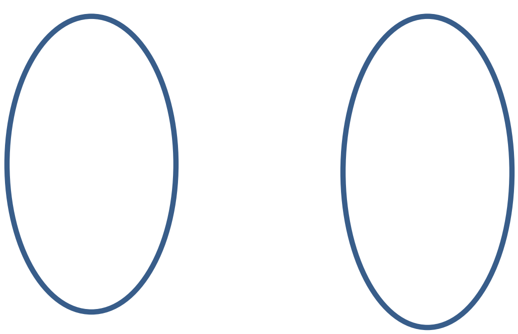
For CANADIAN tourists visiting THE US, temperature data might seem a bit unusual. So a simplified "rule of thumb" for converting a temperature in degrees Fahrenheit into degrees Celsius is …………

(a) Copy and complete the table using the visitor's rule.

°F	10	30	50	70	90	110
°C						

(b) Graph the relation.





(c) What is the independent variable?

(d) What is the dependent variable?

(e) Does our "temperature conversion rule" define a function? Explain.

(f) Let f represent the rule. What ordered pair, (0,????), belongs to f ?

(g) Let x represent the temperature in degrees Celsius. Write the equation for this rule in function notation.

(h) In the table, f (50) = 10, which corresponds to a point on the graph of y = f (x). What is the x-coordinate of this point? What is its y -coordinate?

H. Key Concepts & Notations

a. The **inverse** of a relation and a function maps each output of the original relation back onto the corresponding input value. The inverse is the "reverse" of the original relation, or function

b. f^{-1} is the name given for the inverse relation.

c. $f^{-1}(x)$ represents the expression for calculating the value of f^{-1}.

d. If $(a,b) \in f$, then $(b,a) \in f^{-1}$.

e. Given a table of values for a function, interchange the independent and dependent variables to get a table for the inverse relation.

f. The domain of *f* is the range of f^{-1} and then range of *f* is the domain of f^{-1}.

g. To determine the equation of the inverse in function notation, interchange *x* and *y* and solve for *y*.

I. Examples – Working with Mappings & Relations

1. For each set of ordered pairs,

i. graph the relationship and its inverse

ii. is the relationship a function? Is the inverse a function? Explain.

(a) $\{(0, 1), (1, 3), (2, 5), (3, 7)\}$ (b) $\{(0, 3), (1, 3), (2, 3), (3, 3)\}$

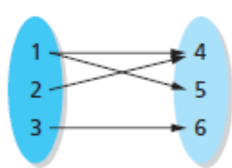
(c) $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$

2. For each of the following,

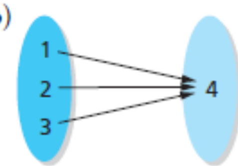
i. draw an arrow diagram for the inverse relationship

ii. state whether or not each inverse defines a function, and justify your answer

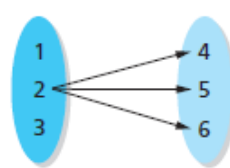
(a)



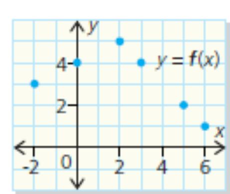
(b)



(c)



The graph of $y = f(x)$ is shown.



i. State the domain and range of *f*.

ii. Draw an arrow diagram for f^{-1}.

iii. Evaluate.

(a) $f(2)$ (b) $f(4)$ (c) $f^{-1}(1)$ (d) $f^{-1}(4)$

iv. Graph $y = f^{-1}(x)$.

v. Is f^{-1} a function? Explain.

vi. State the domain and range of f^{-1}.

J. Examples – Working with Linear Relations

Example 3

The table shows all of the ordered pairs belonging to function g.

x	y
1	5
2	7
3	9
4	11
5	13

(a) Determine $g(x)$.

(b) Write the table for the inverse relation.

(c) Evaluate $g(5)$.

(d) Evaluate $g^{-1}(5)$.

(e) What are the coordinates of the point that corresponds to $g^{-1}(5)$ on the graph of g^{-1}?

(f) What are the coordinates of the point on the graph of g that corresponds to $g^{-1}(5)$?

(g) Determine $g^{-1}(x)$.

Example 5

A relation is $h(x) = -4x + 6$, where $\{x \mid -2 \le x \le 3, x \in \mathrm{R}\}$.

(a) Sketch the graph of $y = h(x)$.

(b) Sketch the graph of $y = h^{-1}(x)$.

(c) State the domain and range of h.

(d) State the domain and range of h^{-1}.

(e) Are h and h^{-1} functions? Explain.

13. Communication: An electronics store pays its employees by commission. The relation $p(s) = 100 + 0.05s$ is used to find an employee's weekly pay, p, in dollars, where s represents the employee's weekly sales in dollars.

(a) Describe the function as a rule.

(b) Determine $p^{-1}(s)$.

(c) Describe the inverse function as a rule.

(d) Describe a situation where the employee might use the inverse function.

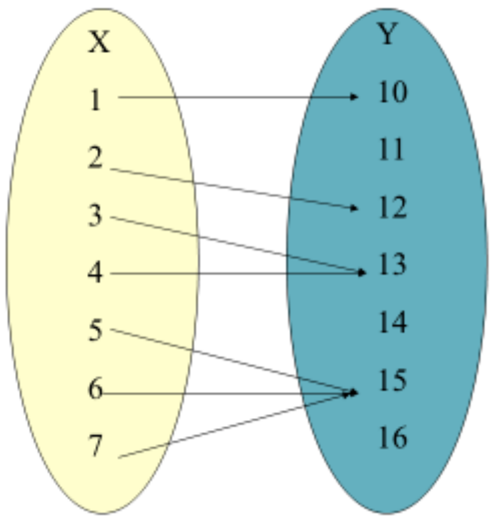
(e) State a reasonable domain and range for p^{-1}.

FURTHER EXAMPLES

(F) Examples

Determine:

(a) Domain of f(x)
(b) Range of f(x)
(c) Mapping of inverse
(d) Domain of inverse
(e) Range of inverse
(f) Is the inverse a function?



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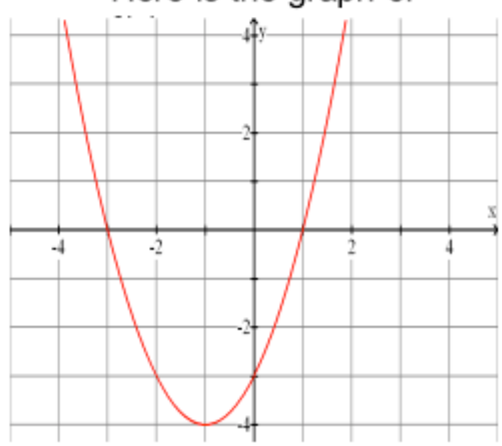
(F) Examples

- Consider a graph of the following data:
- 1. State do domain and range of *f*
- 2. Evaluate f(-2), f(0), $f^{-1}(1)$, $f^{-1}(-2)$
- 3. Graph the inverse
- 4. Is the inverse a function?
- 5. State the domain and range of $f^{-1}(x)$

- Here is the graph of



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FURTHER EXAMPLES

(F) Examples

- ex . If an object is dropped from a height of 80 m, its height above the ground in meters is given by $h(t) = -5t^2 + 80$
- 1. Graph the function
- 2. Find and graph the inverse
- 3. Is the inverse a function
- 4. What does the inverse represent?
- 5. After what time is the object 35 m above the ground?
- 6. How long does the object take to fall?

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(F) Examples

- Ex. Determine the equation for the inverse of $y = 4x - 9$. Draw both graphs and find the D and R of each.
- Ex. Determine the equation for the inverse function of $y = 2x^2 + 4$. Draw both and find D and R of each.
- Ex. Determine the equation of the inverse function of $y = x^2 + 4x - 5$

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