## A. Lesson Context

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|---|---|---|---|
| BIG PICTURE of this UNIT: | • What do we mean by the term "function"<br>• How do we work with the concept of "functions" given our prior experience with linear and exponential relations from IM2?<br>• How do we apply the function concept to linear systems? | | |
| CONTEXT of this LESSON: | Where we've been<br><br>Last year, you were introduced to the concept of relations an functions | Where we are<br><br>How do we work with function notations & the multiple representations of functions, both in a mathematical & a contextual context | Where we are heading<br><br>How do we apply the concept of "functions" to linear & exponential relations |

## B. Lesson Objectives

a. Find the domain and range of a relation.
b. Identify if a relation is a function or not.
c. Work with function notation & evaluating functions.
d. Work with function notation in application based problems.

## C. Fast Five (Skills Review Focus)

The symbols that make up this notation of **f(3) = 7** communicate INFORMATION

| f | 3 | 7 |
|---|---|---|
| | | |

The information being communicated by these "symbols" can also be PRESENTED in ALTERNATE WAYS:

| (i) ordered pair | (ii) mapping | (iii) graph |
|---|---|---|
| | | |

To show the RELATIONSHIP between the INPUT VALUES & OUTPUT VALUES, we can use EQUATIONS. Write at least 3 different equations for which f(3) = 7

## D. REVIEW CONCEPT from IM2: Function Basics

### a. Relations:

i. A "relation" is just a relationship between sets of information;
ii. A relation refers to a set of input and output values, usually represented in ordered pairs
iii. A relation is simply a set of ordered pairs.

### b. Functions:

i. A function is a "well-behaved" relation ➔ When we say that a function is "a well-behaved relation", we mean that, given a starting point, we know exactly where to go; given an *x*, we get only and exactly one *y*.
ii. Function is a relation in which each element of the domain is paired with exactly one element of the range.
iii. A function is a set of ordered pairs in which each x-element has only ONE y-element associated with it.
iv. A function is a rule that takes an input, does something to it, and gives a unique corresponding output.

### c. Notations:

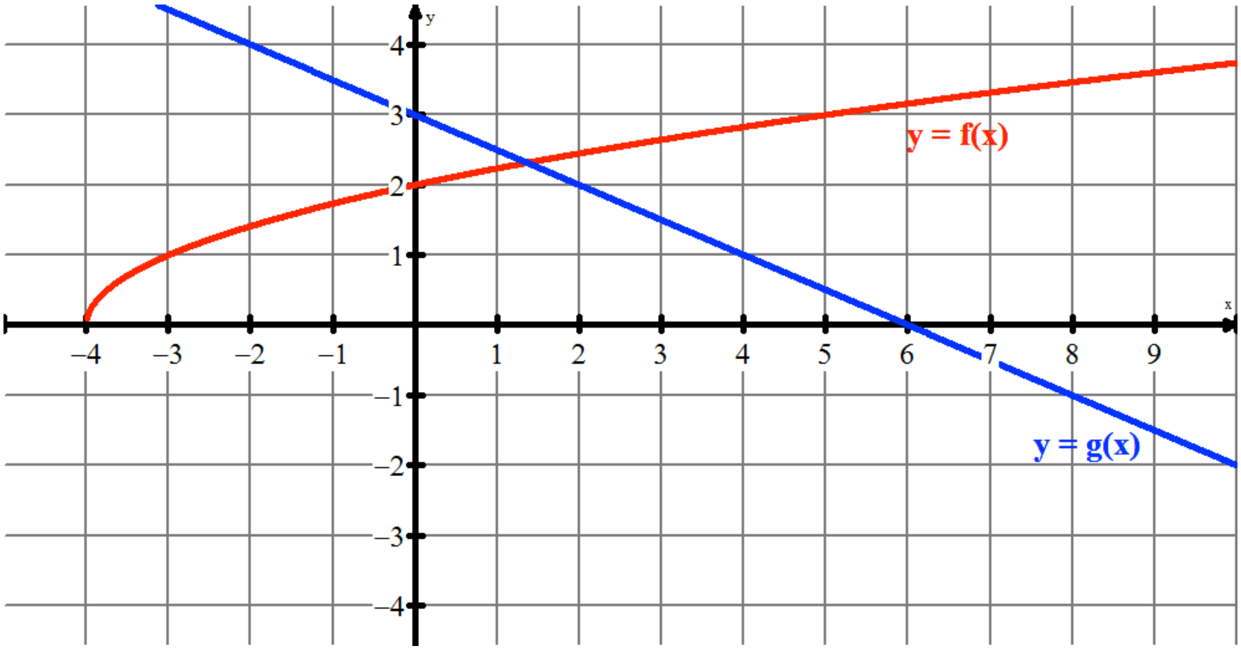
i. Rather than writing linear equations in the typical $y = mx + b$ format, we will now write them in function notation ➔ as $f(x) = mx + b$ where "*f*" simply refers to the function name and the *x* refers to the input and f(x) (or y) refers to the output
ii. Evaluate ➔ if $f(x) = 2x + 4$, then we can evaluate f(3) as ……
iii. Solve ➔ if $f(x) = 2x + 4$, then we can solve $12 = f(x)$ as …..

### d. Understanding Domain and Range:

i. The set of all the starting points is called "the domain" and the set of all the ending points is called "the range."
ii. The domain is what you start with; the range is what you end up with.
iii. The domain is the x's; the range is the y's

## E. Examples ➔ Working with Graphs

Here is a graph showing two functions, y = f(x) and y = g(x), determine each of the following:




(a) Determine f(0)

(b) Determine f(-3)

(c) Determine the value of x if f(x) = 3

(d) State the domain and range of y = f(x)

(a) Determine g(4)

(b) Determine g(-1)

(c) Determine the value of x if g(x) = -1

(d) State the domain and range of y = g(x)

(e) HL EXTENSION Questions:

(i) Determine the value of g(0) – f(0)

(ii) Estimate the value of g(7) – f(7)

(iii) Determine the value of g(5) – g(4)

(iv) Determine the value of the expression $\frac{g(4)-g(2)}{4-2}$? What is the meaning of this value?

(iii) How would you solve a question like "where is g(x) > f(x)"?

(iii) What does the question "Solve for x if f(x) = g(x)" mean?

## F. Further Examples → Working with Algebraic Expressions

Consider these three functions → $f(x) = x^2 - 3x$; $g(x) = 1 - 2x$; $h(x) = \frac{1}{4}(2)^x + 1$ as you answer the following questions:

1. Evaluate f(4), g(4) and h(4)

2. Evaluate f(0), g(0) and h(0) → what is the significance of these values?

3. Evaluate h(-1) as well as h(-4)

4. Show that f(2) > g(2) and explain what this means about the graphs of f(x) and g(x) at x = 2.

5. State the range of y = g(x) if the domain of g(x) were $\{x \in R \mid -2 \leq x < 5\}$

6. HL Extension → Determine f(c + 2) – g(c + 2) as well as h(c + 2) (and simplify the resultant expression
7. HL Extension → GRAPHIC ANALYSIS using ALGEBRA → Determine the range of y = f(x) and y = h(x).
8. HL EXTENSION:
    a. Solve the equation f(x) = g(x) for x.
    b. Determine the value of the difference quotient $\frac{g(2+h)-g(2)}{(2+h)-2}$ and explain its significance.
    c. What happens to the values of h(x) as x values get more and more negative?
    d. Solve the inequality h(x) < 2.

## G. Working with Linear Functions in Context

Since you are studying biological sciences in science this year, here is a biology example. The snowy tree cricket (*Oecanthus fultoni*) has a chirp rate that increases as the air temperature increases. For this type of cricket, the relationship is so reliable that we can use the chirp rate to estimate the temperature. We simply count the number chirps in 15 seconds and add 40 to get the temperature in degrees Fahrenheit. For example, if the cricket chirps 20 times in 15 seconds, then an estimate for the temperature is 20 + 40 = 60°F

### VERBAL (V)

Our function will be defined by the following verbal description ➔ We simply count the number chirps in 15 seconds and add 40 to get the temperature in degrees Fahrenheit

Define the variables that you should be using

n ➔ chirps PER MINUTE

T(n) ➔ Temperature in °F

### NUMERIC (N) - Table of Values

| n | T(n) |
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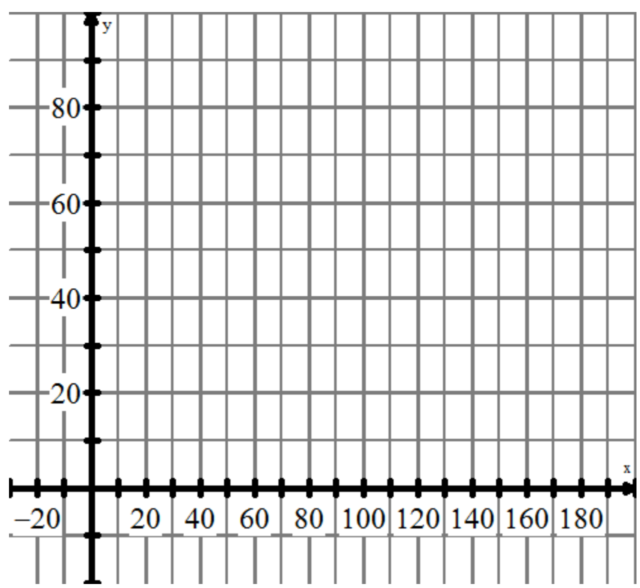
### ALGEBRAIC (A)

### GRAPHIC (G)




### Connection to Functions

Evaluate T(44) ➔

Solve T(n) = 100 ➔

State the domain and a reason for it: ➔

State the range and a reason for it: ➔

## H. Further Examples

### VERBAL (V) & ALGEBRAIC (A)

The population of rabbits in a forest can be modelled by the equation $P(m) = -\frac{1}{4}(m-4)^2 + 25$ where m represents the month of the year (where m = 0 represents January 1st and m = 1 represents February 1st) and P represents the population in hundreds of rabbits

### NUMERIC (N) - Table of Values

| m | P(m) |
|---|---|
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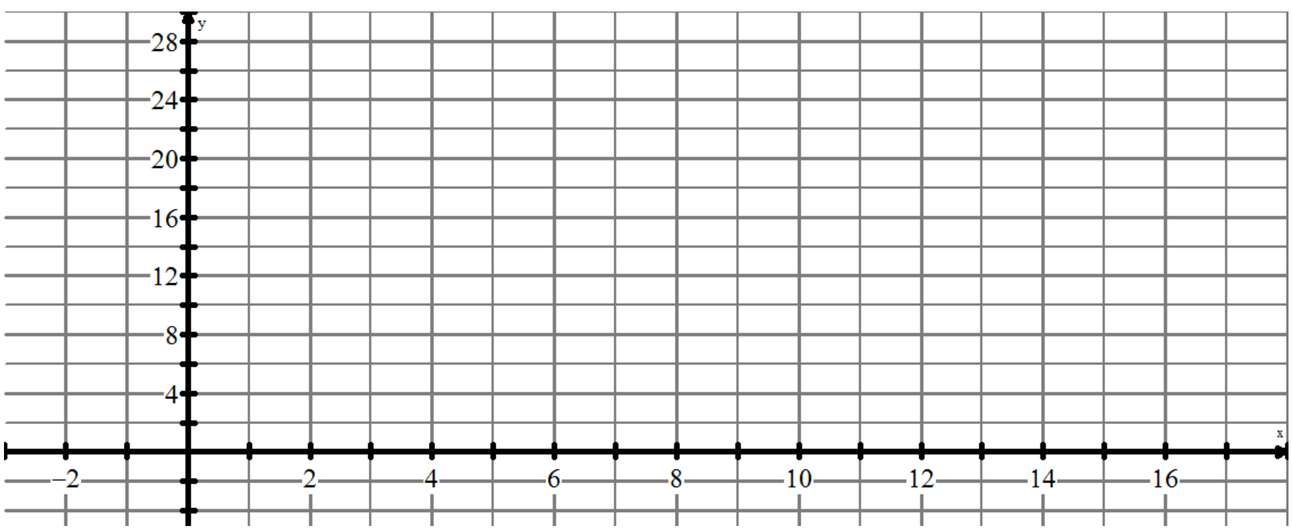
### GRAPHIC (G)




### Connection to Functions

Evaluate P(7) ➔

Solve P(m) = 16 ➔

When is the population of rabbits at a maximum?

When is the population decreasing? Why might that be?

State the domain and a reason for it: ➔

State the range and a reason for it: ➔