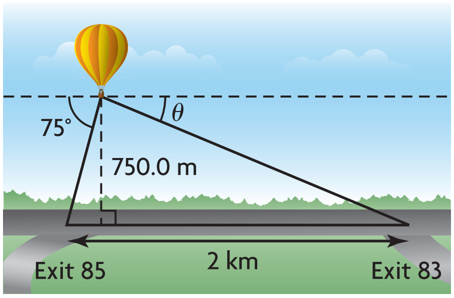
8.5 Solving Acute Triangle Problems

YOU WILL NEED

- ruler

GOAL

Solve problems using the primary trigonometric ratios and the sine and cosine laws.



LEARN ABOUT the Math

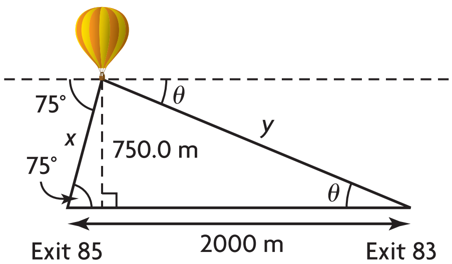
Reid's hot-air balloon is 750.0 m directly above a highway. When Reid is looking west, the angle of depression to Exit 85 is 75°. Exit 83 is located 2 km to the east of Exit 85.

? What is the angle of depression to Exit 83 when Reid is looking east?

EXAMPLE 1 Solving a problem using an acute triangle model

Determine the angle of depression, to the nearest degree, from the balloon to Exit 83.

Vlad's Solution



The ground and the horizontal are parallel, so the alternate angles are equal. The angle of elevation to the balloon at Exit 83 equals θ, the angle of depression I needed to determine. The distance between the exits is 2 km, or 2000 m.

I labelled the unknown sides of the large triangle x and y. If I could determine both x and y, then I could calculate θ using the sine ratio.

$$\sin 75° = \frac{750.0}{x}$$
$$x \sin 75° = 750.0$$
$$x = \frac{750.0}{\sin 75°}$$
$$x \doteq 776.46$$

In the right triangle that contains the 75° angle, x is the hypotenuse and the 750.0 m side is the opposite side. I used the sine ratio to write an equation. Then I solved for x.

$$y^2 = x^2 + 2000^2 - 2(x)(2000)\cos 75°$$
$$y^2 = 776.46^2 + 2000^2 - 2(776.46)(2000)\cos 75°$$
$$y^2 = 602\,890.1316 + 4\,000\,000 - 803\,850.543$$
$$y^2 = 3\,799\,039.589$$
$$y = \sqrt{3\,799\,039.589}$$
$$y \doteq 1949.11$$

I now knew the lengths of two sides in the large acute triangle and the angle between them. So, I was able to use the cosine law to determine y.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{750.0}{1949.11}$$
$$\sin\theta \doteq 0.3848$$

In the right triangle that contained θ, I knew the opposite side, 750.0 m, and the hypotenuse, y. With these values, I was able to determine the value of $\sin\theta$.

$$\theta = \sin^{-1}(0.3848)$$
$$\theta \doteq 22.6°$$

I used the inverse sine to calculate θ.

The angle of depression from the balloon to Exit 83 is about 23°.

I rounded my answer to the nearest degree.

Reflecting

A. Why do you think Vlad started by using the right triangle that contained x instead of the right triangle that contained y?

B. Vlad used the cosine law to determine y. Could he have used another strategy to determine y? Explain.

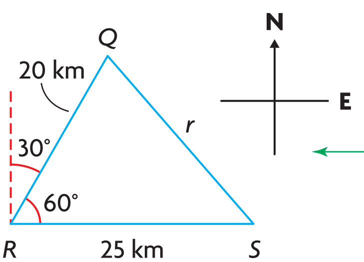
C. Could Vlad have calculated the value of θ using the sine law? Explain.

APPLY the Math

EXAMPLE 2 Solving a problem that involves directions

The captain of a boat leaves a marina and heads due west for 25 km. Then the captain adjusts the course of his boat and heads N30°E for 20 km. How far is the boat from the marina?

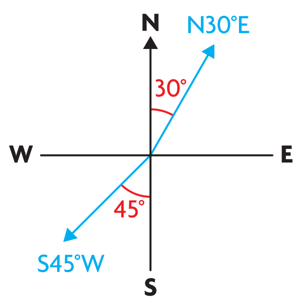
Audrey's Solution



I drew a diagram to represent this situation. The directions north and west are perpendicular to each other. Since N30°E means that the boat travels along a line 30° east of north, I was able to determine $\angle QRS$ by subtracting 30° from 90°.

Communication *Tip*

Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.



$$r^2 = s^2 + q^2 - 2(s)(q)\cos R$$
$$r^2 = 20^2 + 25^2 - 2(20)(25)\cos 60°$$
$$r^2 = 525$$
$$r = \sqrt{525}$$
$$r \doteq 22.9$$

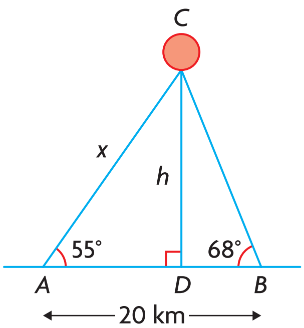
To determine r, I used the cosine law.

The boat is about 23 km from the marina.

EXAMPLE 3 Solving a problem using acute and right triangles

A weather balloon is directly between two tracking stations. The angles of elevation from the two tracking stations are 55° and 68°. If the tracking stations are 20 km apart, determine the altitude of the weather balloon.

Marnie's Solution



I drew a diagram. I did not have enough information about $\triangle ADC$ to calculate h, but I did have enough information about $\triangle ACB$ to calculate x. I reasoned that if I could determine the hypotenuse, x, in the right triangle that contained the 55° angle, then I could use a primary trigonometric ratio to calculate the altitude, h, of the balloon.

$$\angle ACB = 180° - 55° - 68°$$
$$= 57°$$

I knew that the angles in $\triangle ACB$ added up to 180°.

$$\frac{x}{\sin 68°} = \frac{20}{\sin 57°}$$
$$x = \sin 68°\left(\frac{20}{\sin 57°}\right)$$
$$x \doteq 22.1$$

I knew $\angle ACB$ and the side opposite it in $\triangle ACB$. I also knew $\angle B$, which is the angle opposite side x.

I used the sine law to write a proportion that related these values. Then I solved for x.

$$\sin A = \frac{h}{x}$$
$$\sin 55° = \frac{h}{22.1}$$
$$(22.1)(\sin 55°) = h$$
$$18.1 \doteq h$$

$\triangle ACD$ is a right triangle in which h is opposite the 55° angle and x is the hypotenuse.

I used the sine ratio to write an equation. Then I solved for h.

The altitude of the weather balloon is about 18 km.

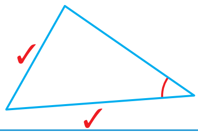
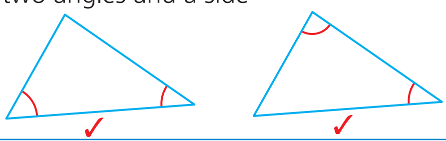
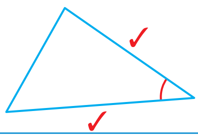
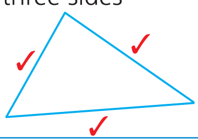
In Summary

Key Ideas

- If a real-world problem can be modelled using an acute triangle, the sine law or cosine law, sometimes along with the primary trigonometric ratios, can be used to determine unknown measurements.
- Drawing a clearly labelled diagram makes it easier to select a strategy for solving the problem.

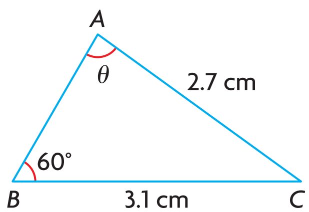
Need to Know

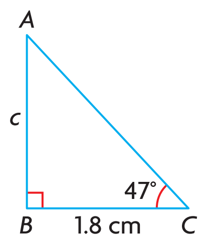
- To decide whether you need to use the sine law or the cosine law, consider the information given about the triangle and the measurement to be determined.

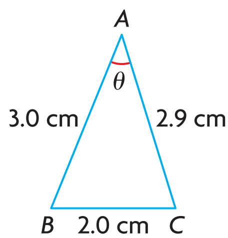
Information Given	Measurement To Be Determined	Use
two sides and the angle opposite one of the sides 	angle	sine law
two angles and a side 	side	sine law
two sides and the contained angle 	side	cosine law
three sides 	angle	cosine law

CHECK Your Understanding

1. Explain how you would determine the measurement of the indicated angle or side in each triangle.

a) 

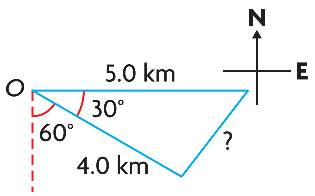
b) 

c) 

2. Use the strategies you described to determine the measurements of the indicated angles and sides in question 1.

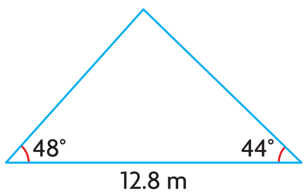
PRACTISING

3. The angle between two equal sides of an isosceles triangle is 52°.
K Each of the equal sides is 18 cm long.
   a) Determine the measures of the two equal angles in the triangle.
   b) Determine the length of the third side.
   c) Determine the perimeter of the triangle.



4. A boat leaves Oakville and heads due east for 5.0 km as shown in the diagram at the left. At the same time, a second boat travels in a direction S60°E from Oakville for 4.0 km. How far apart are the boats when they reach their respective destinations?

5. A radar operator on a ship discovers a large sunken vessel lying flat on the ocean floor, 200 m directly below the ship. The radar operator measures the angles of depression to the front and back of the sunken ship to be 56° and 62°. How long is the sunken ship?



6. The base of a roof is 12.8 m wide as shown in the diagram at the left. The rafters form angles of 48° and 44° with the horizontal. How long, to the nearest tenth of a metre, is each rafter?

7. A flagpole stands on top of a building that is 27 m high. From a point on the ground some distance away, the angle of elevation to the top of the flagpole is 43°. The angle of elevation to the bottom of the flagpole is 32°.
   a) How far is the point on the ground from the base of the building?
   b) How tall is the flagpole?

8. Two ships, the *Albacore* and the *Bonito*, are 50 km apart. The *Albacore*
A is N45°W of the *Bonito*. The *Albacore* sights a distress flare at S5°E. The *Bonito* sights the distress flare at S50°W. How far is each ship from the distress flare?

Career Connection

Pilots and flight engineers transport people, goods, and cargo. Some test aircraft, monitor air traffic, rescue people, or spread seeds for reforesting.

9. Fred and Agnes are 520 m apart. As Brendan flies overhead in an airplane, they measure the angle of elevation of the airplane. Fred measures the angle of elevation to be 63°. Agnes measures it to be 36°. What is the altitude of the airplane?

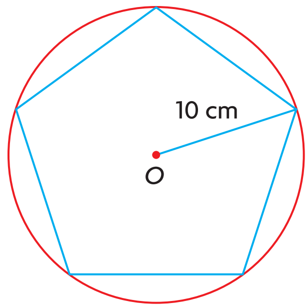
10. The *Nautilus* is sailing due east toward a buoy. At the same time, the *Porpoise* is approaching the buoy heading N42°E. If the *Nautilus* is 5.4 km from the buoy and the *Porpoise* is 4.0 km from the *Nautilus*, on a heading of S46°E, how far is the *Porpoise* from the buoy?

11. Two support wires are fastened to the top of a satellite dish tower from points *A* and *B* on the ground, on either side of the tower. One wire is 18 m long, and the other wire is 12 m long. The angle of elevation of the longer wire to the top of the tower is 38°.

a) How tall is the satellite dish tower?

b) How far apart are points *A* and *B*?

12. A regular pentagon is inscribed in a circle with radius 10 cm as shown **T** in the diagram at the right. Determine the perimeter of the pentagon.



13. Ryan is in a police helicopter 400 m directly above a highway. When he looks west, the angle of depression to a car accident is 65°. When he looks east, the angle of depression to the approaching ambulance is 30°.

a) How far away is the ambulance from the scene of the accident?

b) The ambulance is travelling at 80 km/h. How long will it take the ambulance to reach the scene of the accident?

14. The radar screen in an air-traffic control tower shows that two airplanes are at the same altitude. According to the range finder, one airplane is 100 km away, in the direction N60°E. The other airplane is 160 km away, in the direction S50°E.

a) How far apart are the airplanes?

b) If the airplanes are approaching the airport at the same speed, which airplane will arrive first?

15. In a parallelogram, two adjacent sides measure 10 cm and 12 cm. The shorter diagonal is 15 cm. Determine, to the nearest degree, the measures of all four angles in the parallelogram.

16. Create a real-life problem that can be modelled by an acute triangle. **C** Then describe the problem, sketch the situation in your problem, and explain what must be done to solve it.

Extending

17. From the top of a bridge that is 50 m high, two boats can be seen anchored in a marina. One boat is anchored in the direction S20°W, and its angle of depression is 40°. The other boat is anchored in the direction S60°E, and its angle of depression is 30°. Determine the distance between the two boats.

18. Two paper strips, each 5 cm wide, are laid across each other at an angle of 30°, as shown at the right. Determine the area of the overlapping region. Round your answer to the nearest tenth of a square centimetre.