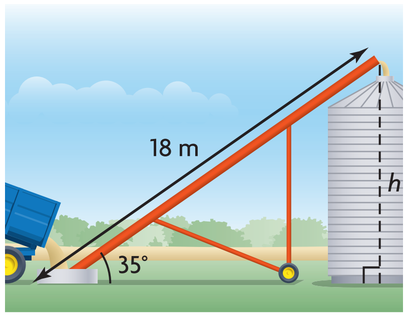
7.5 Solving Right Triangles

GOAL

Use primary trigonometric ratios to calculate side lengths and angle measures in right triangles.

LEARN ABOUT the Math

A farmers' co-operative wants to buy and install a grain auger. The auger would be used to lift grain from the ground to the top of a silo. The greatest angle of elevation that is possible for the auger is 35°. The auger is 18 m long.

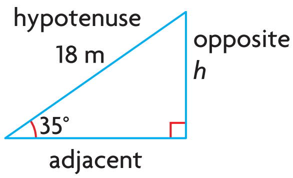


? What is the maximum height that the auger can reach?

EXAMPLE 1 **Solving a problem** for a side length using a trigonometric ratio

Calculate the maximum height that the auger can reach.

Hong's Solution



I drew a diagram to model the problem. The height is the length of the side that is opposite the 35° angle. I named the other sides relative to the 35° angle.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 35° = \frac{h}{18}$$

Because I knew the length of the hypotenuse, I used the sine ratio. The sine of 35° equals the opposite side, or height, divided by the hypotenuse.

$$18\ (\sin 35°) = 18\left(\frac{h}{18}\right)$$

$$18\ (\sin 35°) = h$$

$$10 \doteq h$$

I multiplied both sides by 18 and evaluated 18 (sin 35°) using a calculator. I rounded my answer using the degree of accuracy in the other measures.

The maximum height that the auger can reach is about 10 m.

Tech | *Support*

For help using a TI-83/84 graphing calculator to calculate trigonometric ratios, see Appendix B-12. If you are using a TI-*n*spire, see Appendix B-48.

Reflecting

A. If the height of the grain auger is increased, what happens to the sine, cosine, and tangent ratios for the angle of elevation? Explain.

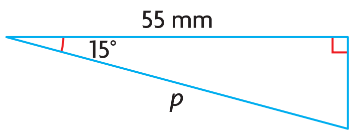
B. Why can you use either the sine ratio or the cosine ratio to calculate the maximum height?

C. Explain why Hong might have chosen to use the sine ratio.

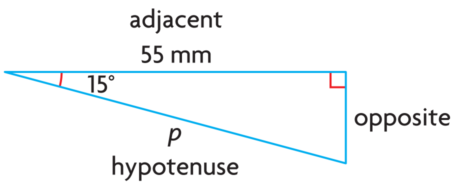
APPLY the Math

EXAMPLE 2 Connecting the cosine ratio with the length of the hypotenuse

Determine the length of p.



Mandy's Solution



The 55 mm side is adjacent to the 15° angle. I named the rest of the sides in the triangle relative to the 15° angle.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 15° = \frac{55}{p}$$

Because I knew the adjacent side and had to determine the hypotenuse, I used the cosine ratio. The cosine of 15° equals the adjacent side divided by the hypotenuse.

$$p(\cos 15°) = p\left(\frac{55}{p}\right)$$
$$p(\cos 15°) = 55$$
$$\frac{p(\cos 15°)}{\cos 15°} = \frac{55}{\cos 15°}$$
$$p = \frac{55}{\cos 15°}$$
$$p \doteq 57$$

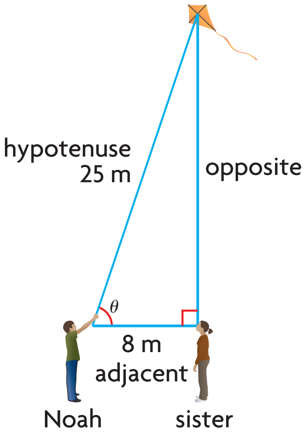
I multiplied both sides by p. Then I divided both sides by cos 15° to solve for p. I rounded my answer to the nearest millimetre.

The length of p is about 57 mm long.

EXAMPLE 3 Connecting the cosine ratio with an angle measure

Noah is flying a kite and has released 25 m of string. His sister is standing 8 m away, directly below the kite. What is the angle of elevation of the string?

Jacob's Solution



I drew a right triangle showing Noah, his sister, and the kite. I labelled the triangle with the information that I knew and named the sides of the triangle relative to the angle of elevation.

$$\cos\theta = \frac{8}{25}$$

Because I knew the lengths of the adjacent side and the hypotenuse, I used the cosine ratio.

$$\theta = \cos^{-1}\left(\frac{8}{25}\right)$$

I used the inverse cosine to determine the angle.

$$\theta \doteq 71°$$

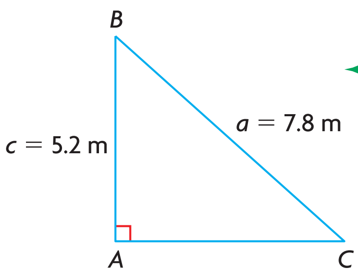
I rounded my answer to the nearest degree.

The angle of elevation of the string is about 71°.

EXAMPLE 4 Selecting a trigonometric strategy to solve a triangle

Solve $\triangle ABC$, given $\angle A = 90°$, $a = 7.8$ m, and $c = 5.2$ m.

Chloe's Solution



I drew the triangle and labelled it with the measurements that I knew.

Communication *Tip*

To solve a triangle means to determine all unknown angle measures and side lengths.

$$\cos B = \frac{c}{a}$$

I started with $\angle B$. Since side c is adjacent to $\angle B$ and side a is the hypotenuse, I used the cosine ratio.

$$\cos B = \frac{5.2}{7.8}$$

$$\angle B = \cos^{-1}\left(\frac{5.2}{7.8}\right)$$

$$\angle B \doteq 48°$$

I rounded my answer to the nearest degree.

$$\angle C \doteq 180° - 90° - 48°$$
$$\angle C \doteq 42°$$

I knew the sum of the angles in a triangle is 180°. I used this to determine $\angle C$.

$$b^2 + c^2 = a^2$$
$$b^2 + 5.2^2 = 7.8^2$$
$$b^2 + 27.04 = 60.84$$
$$b^2 = 60.84 - 27.04$$
$$b^2 = 33.80$$
$$b = \sqrt{33.80}$$
$$b \doteq 5.8$$

I used the Pythagorean theorem to solve for b. I could have used the sine or tangent ratios instead.

In $\triangle ABC$, $\angle B \doteq 48°$, $\angle C \doteq 42°$, and $b \doteq 5.8$ m.

In Summary

Key Idea

- Trigonometric ratios can be used to calculate unknown side lengths and unknown angle measures in a right triangle. The ratio you use depends on the information given and the quantity you need to calculate.

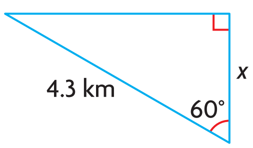
Need to Know

- To determine the length of a side in a right triangle using trigonometry, you need to know the length of another side and the measure of one of the acute angles.
- To determine the measure of one of the acute angles in a right triangle using trigonometry, you need to know the lengths of two sides.

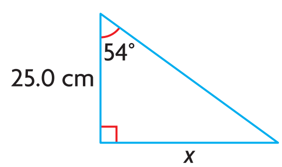
CHECK Your Understanding

1. Solve for x, to one decimal place, using the indicated trigonometric ratio.

a) cosine

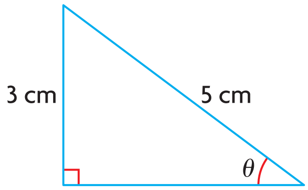


b) tangent

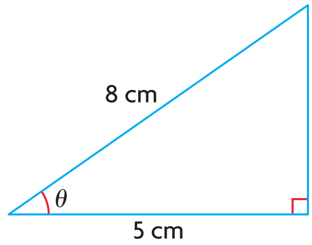


2. Determine the value of θ, to the nearest degree, in each triangle.

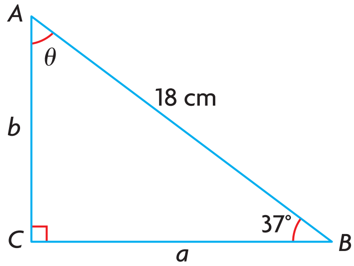
a)



b)



3. Solve $\triangle ABC$.



PRACTISING

4. Solve for $\angle A$ to the nearest degree.

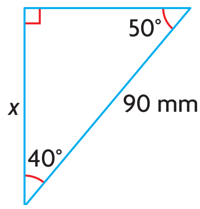
a) $\sin A = 0.9063$ **b)** $\cos A = \frac{4}{5}$

5. For each triangle,

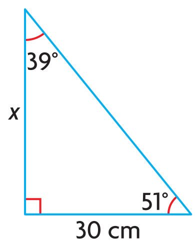
K **i)** state two trigonometric ratios you could use to determine x

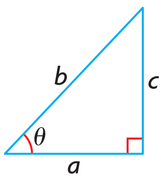
ii) determine x to the nearest unit

a)



b)



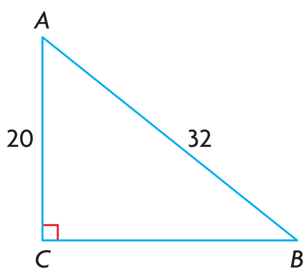


6. For each pair of side lengths, calculate the measure of θ to the nearest degree for the triangle at the left.

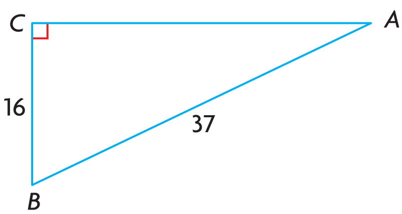
a) $a = 10$ and $c = 10$ **b)** $b = 12$ and $c = 6$ **c)** $a = 9$ and $b = 15$

7. Using trigonometry, calculate the measures of $\angle A$ and $\angle B$ in each triangle. Round your answers to the nearest degree.

a)



b)

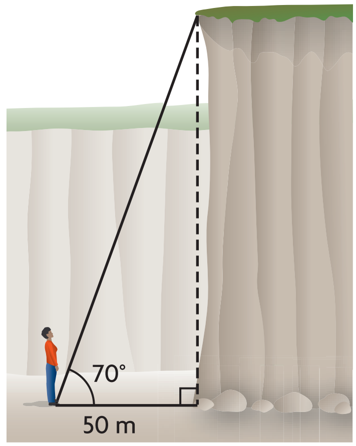


8. Calculate the measure of the indicated angle, to the nearest degree, in each triangle.

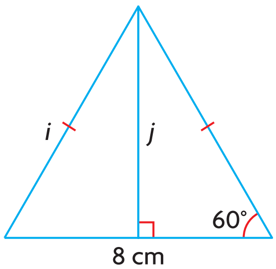
a) In $\triangle ABC$, $\angle C = 90°$, $a = 11.3$ cm, and $b = 9.2$ cm. Calculate $\angle A$.

b) In $\triangle DEF$, $\angle D = 90°$, $d = 8.7$ cm, and $f = 5.4$ cm. Calculate $\angle F$.

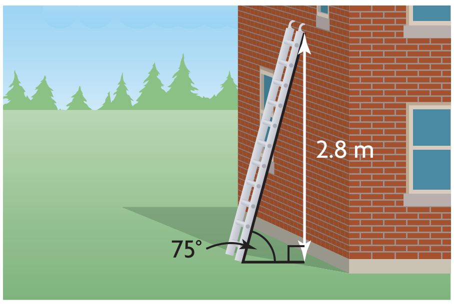
9. Janice is getting ready to climb a steep cliff. She needs to fasten herself to a rope that is anchored at the top of the cliff. To estimate how much rope she needs, she stands 50 m from the base of the cliff and estimates that the angle of elevation to the top is 70°. How high is the cliff?



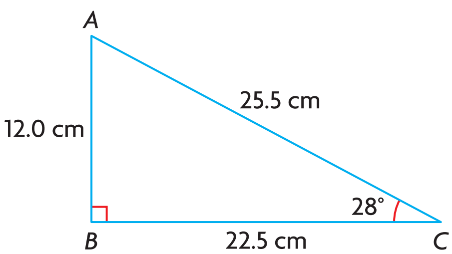
10. Solve for i and j.
T



11. A ladder leans against a wall, as shown. How long is the ladder, to the nearest tenth of a metre?
A



12. Kelsey made these notes about $\triangle ABC$. Determine whether each answer is correct, and explain any errors.
C



a) $\sin A = \frac{12.0}{25.5}$

b) $\angle A = 62°$

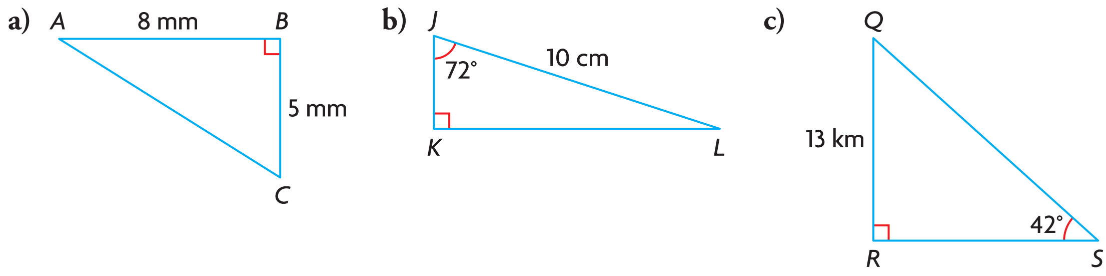
c) $\cos C = \frac{25.5}{22.5}$

d) $\tan A = 1.875$

e) $\sin C = \frac{24}{51}$

f) $\tan C = 0.53$

13. Solve each triangle. Round the measure of each angle to the nearest degree. Round the length of each side to the nearest unit.



14. For a ladder to be stable, the angle that it makes with the ground should be no more than 78° and no less than 73°.

a) If the base of a ladder that is 8.0 m long is placed 1.5 m from a wall, will the ladder be stable? Explain.

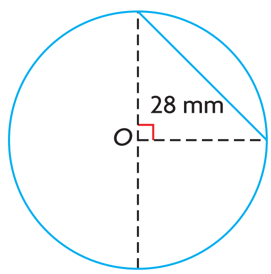
b) What are the minimum and maximum safe distances from the base of the ladder to the wall?

15. a) Create a mind map that shows the process of choosing the correct trigonometric ratio to determine an unknown measure in a right triangle.

b) Does the process differ depending on whether you are solving for a side length or an angle measure? Explain.

Extending

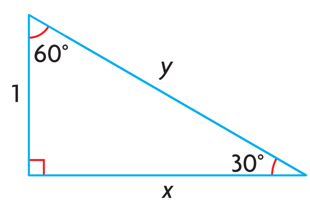
16. Determine the diameter of the circle, if *O* is the centre of the circle.



17. a) Determine the exact value of *x* in the triangle at the right using trigonometry.

b) Determine the exact value of *y* using the Pythagorean theorem.

c) Determine the sine, cosine, and tangent ratios of both acute angles. What do you notice?



18. a) Draw a right isosceles triangle.

b) Calculate the sine and cosine ratios for one of the acute angles. Explain your results.