CAC CAIRO AMERICAN COLLEGE

Name: _____Date : _____

IM2 UNIT 5 TEST V2 – Quadratic Functions

Teacher: Mr. Santowski, Mr. Smith and Mr. Rawlings

Score:_____/

PART 1 – THE BASICS

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided the answer is supported by working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer

1. Use ***algebraic methods*** to solve the following equations:

(12 marks)

(a) $3x(x-2)=0$

(b) $(x+5)(2x-3)=0$

(d) $3x^2-4-x=0$

(b) $(x-1)(x-4)=18$

2. Given the quadratic equation $0 = 4 - 11x - 15x^2$.

(6 marks)

a. State the values of a, b and c.

b. Use the quadratic formula to solve $0 = 4 - 11x - 15x^2$.

c. Verify your answers using a graph. Sketch a copy of the graph and clearly label where in the graph the solutions are shown.

d. Given your work in Part b and Part c, write the quadratic equation $f(x) = 4 - 11x - 15x^2$ in factored form

3. A quadratic function, $f(x)$, has one zero/x-intercept at x = 4 and the vertex is located at (7,-4).

(2 marks)

a. Determine the location of the second zero or x-intercept.

b. HENCE, determine the solution to the equation $f(x) = 0$.

4. Tuka is trying to solve the equation $2x^2 + 3x + 7 = 0$. She decides to use the quadratic formula and once she has started this algebraic solution, she realizes that there is NO solution to the equation.

(4 marks)

a. Explain/show how her algebraic solution informs Tuka that there is NO solution.

b. Explain how Tuka can use a GRAPH to confirm her algebraic work and conclusions.

5. If the vertex of a parabola is (4,-3) and the value of ***a*** is -2, the parabola has two x-intercepts. TRUE or FALSE? Explain your reasoning OR show your workings.

(2 marks)

6. The parabola y = -2(x + 4)(3 – x) has a maximum value. TRUE or FALSE? Explain your reasoning OR show your workings.

(2 marks)

7. You are now going to solve the equation $3x + 7 = 2x^2 + 2x + 1$.

   a. Provide an **algebraic** solution to this problem (factoring/Quad formula).

(4 marks)

   b. Kyumin would like to solve this problem with a graph and so decides to find an intersection point. Use your TI-84 to find the intersection point(s) of the line $f(x) = 3x + 7$ and the parabola $g(x) = 2x^2 + 2x + 1$. (PROVIDE a sketch as "evidence" of your work)

(2 marks)

   c. Are the answers to Part a and Part b the same? Offer an explanation as to why/why not.

(1 mark)

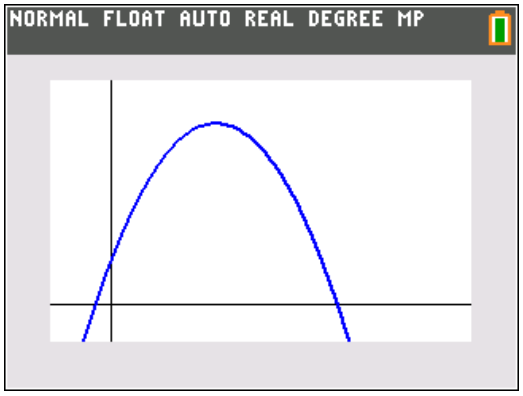
PART 2 – THE PROBLEMS

1. Quadratic functions are often used to model the trajectories of projectiles. Mahmoud is shooting a ball from the first floor balcony of the MS building and the height of the projectile, in meters above the ground, is given as $h(t) = -2t^2 + 21t + 12$ where is t is time in seconds since he released the ball.

(10 marks)

a. Determine vertex and explain what is means in the context of this problem.

(2M)



b. Mahmoud would like to solve the equation $0 = -2t^2 + 21t + 12$. Explain what the SOLUTION(S) to this equation MEAN in this problem.

(2M)

c. For how long was the ball in flight.

(1M)

d. State a domain and range for this height function. Explain your reasoning.

(2M)

e. Determine the time interval during which the height reached was **higher** than 40 meters.

(2M)

f. Yehia decides that he needs an equation of a trajectory that has the SAME time of flight, starts at the same release height, but reaches a HIGHER maximum. Determine an equation of a parabola that Yehia can use, given his requirements (same time of flight but higher maximum). Show your work/explain your thinking.

(1M)

2. We have also used quadratic functions to model business applications. So, the profits of a company in its first 15 months of operations are modeled by the quadratic function $P(m) = -0.5m^2 + 9m - 22.5$, where m is the number of months (where $m = 0$ represents January 1st and so $m = 1.5$ represents mid-February) and $P(m)$ is measured in millions of Egyptian pounds (EGP).

(12 marks)

a. Jewoo rewrites the function $P(m) = -0.5m^2 + 9m - 22.5$ as $P(m) = -\frac{1}{2}(m^2 - 18m + 45)$. How did Jewoo do this?

(1M)

b. Determine in which **month** the company maximizes its profits.

(2M)

c. What are the company's maximum profits?

(1M)

d. Determine when the company "breaks even" (company has NO profit).

(2M)

e. Solve and interpret P(m) < 0 given that the domain is $\{0 \le m \le 15\}$

(2M)

f. For what values of m are the profits DECREASING in our domain of $\{0 \le m \le 15\}$?

(2M)

g. Solve $P(m) = 12$ and interpret.

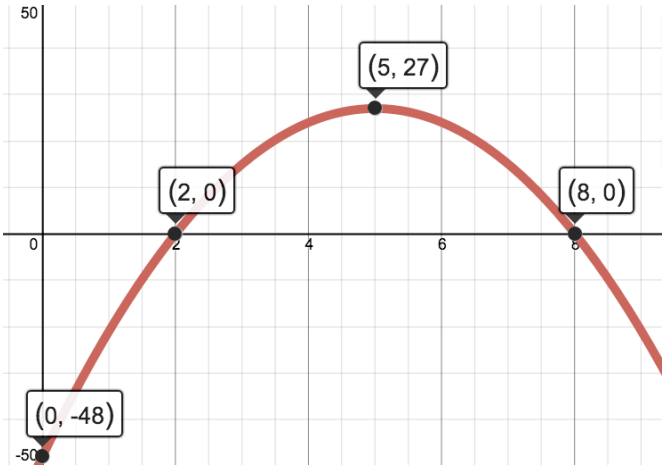
(2M)

3. Part of the skill set you need to be successful in Math is an ability to make connections. So, a graph of parabola is provided below.

(8 marks)

a. Sam knows the equation MUST be $y = -3(x-2)(x-8)$. Explain/show how Sam determined this equation.

(2M)



Calvin now wonders about applying some ideas from SEM 1. In our geometry unit, we looked at moving/translating geometric shapes and Calvin would now like to move this parabola. First, Calvin decides to **move the parabola 4 units to the left**. (Recall: (x, y) → (x – 4, y)).

b. Determine where the x-intercepts and the vertex are now located.

(2M)

c. Does the value of "*a*" in the quadratic equation change when the parabola is moved left by 4? Explain your thinking or reasoning.

(2M)

Aidan now wonders what would happen if the parabola were reflected over the line y = x (Recall the mathematics for this change: (x, y) → (y , x)).

d. Determine where the x-intercepts and the vertex are now located.

(1M)

e. What would the parabola now look like after this transformation? SKETCH the parabola.

(1M)

BONUS: Determine an equation that could be used for this new parabola.