Chapter Review

FREQUENTLY ASKED Questions

- Q: How do you use the coordinates of the vertices of a triangle or quadrilateral to determine what type of figure it is?
- A: To determine whether a triangle is isosceles, equilateral, or scalene, you calculate the side lengths using the distance formula. To determine whether the triangle is a right triangle, you substitute the side lengths into the Pythagorean theorem to see if they work, or you calculate the slopes of the line segments to see if two of the slopes are negative reciprocals.



For a quadrilateral, you determine the length and slope of each line segment that forms a side. Then you compare the lengths to see if there are equal sides, and compare the slopes to see if any sides are parallel or perpendicular.



Study Aid

- See Lesson 2.4, Examples 1 and 2.
- Try Chapter Review Questions 12 to 15.

Study Aid

- See Lesson 2.5, Examples 1 to 3.
- Try Chapter Review Questions 16 to 20.

Q: How can you use the coordinates of vertices to verify properties of triangles, quadrilaterals, or circles?

A: You can use the coordinates of vertices to calculate midpoints and slopes, as well as side lengths in a triangle, lengths of sides or diagonals in a quadrilateral, or lengths of chords in a circle. Then you can use these values to verify properties of the figure.

Q: How do you use coordinates to locate a point that is the same distance from three given points?

A: You draw two line segments that join two pairs of given points. Then you determine the point of intersection of the perpendicular bisectors of these line segments. The point where the perpendicular bisectors intersect (called the circumcentre) is the same distance from the three given points.



Q: How do you calculate the distance from a point to a line?

A: The distance from a point to a line is the perpendicular distance, since this is the shortest possible distance.



To calculate the distance from P to the line in the diagram:

- Determine the equation of a perpendicular line that goes through *P*. To do this, take the negative reciprocal of the slope of the line in the diagram. Then use the coordinates of *P* to determine the *y*-intercept of the perpendicular line.
- Determine the coordinates of the point of intersection of the line in the diagram and the perpendicular line by solving the linear system formed by the two lines.
- Determine the length of the line segment that joins *P* to this point of intersection using the distance formula.

Study **Aid**

- See Lesson 2.7, Example 1.
- Try Chapter Review Question 23.

Study Aid

- See Lesson 2.3, Example 3, and Lesson 2.7, Example 2.
- Try Chapter Review Question 25.

PRACTICE Questions

Lesson 2.1

- On the design plan for a garden, a straight path runs from (-25, 20) to (40, 36). A lamp is going to be placed at the midpoint of the path. Determine the coordinates for the lamp.
- **2.** $\triangle ABC$ has vertices at A(-4, 4), B(-4, -2), and C(2, -2).
 - a) Determine the equation of the median from *B* to *AC*.
 - **b)** Is the median for part a) also an altitude? Explain how you know.
- 3. $\triangle LMN$ has vertices at L(0, 4), M(-5, 2), and N(2, -2). Determine the equation of the perpendicular bisector that passes through MN.

Lesson 2.2

- **4.** Which point is closer to the origin: *P*(−24, 56) or *Q*(35, −43)?
- **5.** A builder needs to connect a partially built house to a temporary power supply. On the plan, the coordinates of the house are (20, 110) and the coordinates of the power supply are (105, 82). What is the least amount of cable needed?
- **6.** $\triangle QRS$ has vertices at Q(2, 6), R(-3, 1), and S(6, 2). Determine the perimeter of the triangle.
- *AXYZ* has vertices at X(1, 6), Y(−3, 2), and Z(9, 4). Determine the length of the longest median in the triangle.

Lesson 2.3

- 8. a) Determine the equation of the circle that is centred at (0, 0) and passes through point (-8, 15).
 - **b)** Identify the coordinates of the intercepts and three other points on the circle.
- **9.** A circle has a diameter with endpoints C(20, -21) and D(-20, 21). Determine the equation of the circle.

10. Determine the equation of this circle.



11. The point (-2, k) lies on the circle x² + y² = 20. Determine the values of k. Show all the steps in your solution.

Lesson 2.4

12. $\triangle ABC$ has vertices as shown. Use analytic geometry to show that $\triangle ABC$ is isosceles.



- 13. A triangle has vertices at A(1, 1), B(-2, -1), and C(3, -2). Calculate the side lengths to determine whether the triangle is isosceles, equilateral, or scalene.
- **14.** Show that the quadrilateral with vertices at J(-1, 1), K(3, 4), L(8, 4), and M(4, 1) is a rhombus.
- **15.** Determine the type of quadrilateral described by the vertices R(-3, 2), S(-1, 6), T(3, 5), and U(1, 1). Show all the steps in your solution.

Lesson 2.5

16. A quadrilateral has vertices at A(-3, 1), B(-5, -9), C(7, -1), and D(3, 3). Show that the midsegments of the quadrilateral form a parallelogram.

- 17. Show that points (10, 10), (-7, 3), and (0, -14) lie on a circle with centre (5, -2).
- **18.** A triangle has vertices at P(-2, 7), Q(-4, 2), and R(6, -2).
 - **a)** Show that $\triangle PQR$ is a right triangle.
 - **b)** Show that the midpoint of the hypotenuse is the same distance from each vertex.
- **19.** a) Show that points (6, 7) and (-9, 2) are the endpoints of a chord in a circle with centre (0, 0).
 - b) A line is drawn through the centre of the circle so that it is perpendicular to the chord. Verify that this line passes through the midpoint of the chord.
- **20. a)** Quadrilateral *JKLM* has vertices as shown. Show that the diagonals of the quadrilateral bisect each other.



- **b)** Make a conjecture about the type of quadrilateral *JKLM* could be.
- **c)** Use analytic geometry to verify your conjecture.

Lesson 2.7

21. $\triangle PQR$ has vertices at P(0, -2), Q(4, 4), and R(-4, 5). Use analytic geometry to determine the coordinates of the orthocentre (the point where the altitudes intersect).



22. $\triangle XYZ$ has vertices at X(0, 1), Y(6, -1), and Z(3, 6). Use analytic geometry to determine the coordinates of the centroid (the point where the medians intersect).



23. A new lookout tower is going to be built so that it is the same distance from three ranger stations. If the stations are at A(-90, 28), B(0, -35), and C(125, 20) on a grid, determine the coordinates of the point where the new tower should be built.



- 24. Predict the type of quadrilateral that is formed by the points of intersection of the lines 3x + y - 4 = 0, 4x - 5y + 30 = 0, y = -3x - 1, and -4x + 5y + 10 = 0. Give reasons for your prediction. Verify that your prediction is correct by solving this problem.
- **25.** A builder wants to run a temporary line from the main power line to a point near his site office. On the site plan, the site office is at S(25, 18) and the main power line goes through points T(1, 5) and U(29, 12). Each unit represents 1 m.
 - a) At what point should the builder connect to the main power line?
 - **b)** What length of cable will the builder need?