DAY 2 - Problem Solving with Quadratic Equations | Unit 6 Lesson 9

(A) Lesson Context

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic equations?		
CONTEXT of this LESSON:	Where we've been In Lessons 8 & 9, you were solving quadratic equations using factoring strategies OR the quadratic formula	Where we are We will now solve quadratic equations IN CONTEXT problems, wherein we ultimately need to use the process of factoring or the quadratic formula	Where we are heading How can I use EQUATIONS to make predictions about parabolas and quadratic data sets & quadratic models

(B) Lesson Objectives:

a. Use the skills of solving quadratic equations in contextual problems with quadratic functions

(C) Modeling with Quadratic Functions – ALL LEVELS

Ex. 1: Mr Santowski runs a clothing business and models how his revenues on sales of denim jeans are related to price changes. He uses the quadratic equation $R = 300 + 20x - x^2$, where R represents his daily revenue in dollars and x represents an increase or decrease in price. (So x = +1 represents a price increase of 1 dollar and x = -2 represents a price decrease of 2 dollars)

a. Determine the price change that will result in maximum revenues. What is the maximum revenue

b. Factor the equation $R = 300 + 20x - x^2$.

c. Solve the equation $0 = 300 + 20x - x^2$ and interpret what the answers mean, given the context.

d. Make a sketch of the relation.

e. Solve the equation $300 = 300 + 20x - x^2$ and interpret what the answers mean, given the context.

f. Solve the equation $375 = 300 + 20x - x^2$ and interpret what the answers mean, given the context

Ex. 2: You will find a picture of my friend who visited Arches National Park a couple of years ago. She is 5 foot 6 inches tall. Determine an equation you can use to model the arch under which she is standing

Ex. 3: Mr. S. can sell 500 apples per week when he charges 50 cents per apple. Through market research, his wife (being smarter than Mr. S of course) knows that for every price increase of 2 cents per apple, he will sell 10 less apples.

a. EXPLAIN what the expression $(500 - 10x)$ would represent in this problem.
b. EXPLAIN what the expression $(0.50 + 0.02x)$ would represent in this problem.
c. What would the variable x represent in the first place?
d. Determine an equation that can you used to model Mr. S.'s expected revenues.
e. What price should he charge to maximize his revenues?
f. What is his maximum revenue?
g. How many price increments are required such that his business has NO revenue?

(C) GREEN LEVEL Practice Problems

1. The cost per hour of running a bus between Burlington and Toronto is modeled by the function $C(x) = 0.0029x^2 - 0.48x + 142$, where *x* is the speed of the bus in kilometres per hour, and the cost, *C*, is in dollars. Determine the most cost-efficient speed for the bus and the cost per hour at this speed.

2. ***(CA)*** The formula for the height, *h* in meters, of an object launched into the air as a function of its time in flight, *t* in seconds, is given by is $h(t) = -\frac{1}{2}gt^2 + v_o t + h_o$, where g represents the acceleration due to gravity which is about 9.8 m/s^2, v_o refers to the launch velocity in m/s and h_o represents the initial launch height in m.

   If a projectile has an initial velocity of 34.3 m/s and is launched 2.1 m above the ground, graphically determine:

   a. the equation that you will enter into the TI-84
   b. the time at which the projectile reaches the maximum height
   c. the maximum height reached by the projectile
   d. h(2)
   e. solve for t if 12 = h(t)
   f. state the domain and range of the relation and explain WHY
   g. the x-intercepts and their significance
   h. the total time of flight of the projectile

3. ***(CI)*** Determine the flight time of a projectile whose height, *h*(t) in meters, varies with time, *t* in seconds, as per the following formula: $h(t) = -5t^2 + 15t + 50$
   a. Determine a reasonable domain for the function. What does it mean in context?
   b. What is the range? What does it mean in context?
   c. Does the projectile attain a height of 70m?
   d. Determine the maximum height of the projectile?
   e. When does the object reach this height?
   f. When does the projectile attain a height of 60 meters?

4. The path of a baseball thrown at a batter by Mr S is modeled by the equation $h(d) = -0.004d^2 + 0.06d + 2$, where h is the height in m and d is the horizontal distance of the ball in meters from the batter.

    a. What is the maximum height reached by the baseball?
    b. What is the horizontal distance of the ball from the batter when the ball reaches its maximum height?
    c. How far from the ground is the ball when I release the pitch?
    d. How high above the ground is the ball when the ball reaches the batter if she stands 20 m from the pitcher

5. You will find a picture of Sydney Harbour Bridge. The bottom "arch" of the bridge is shaped as a parabola. You will need to research some data about the bridge so that you can determine an equation you can use to model the Bridge

(D)BLACK LEVEL Challenge Problems

1. Student council plans to hold a talent show to raise money for charity. Last year, they sold tickets for $11 each and 400 people attended. Student council decides to raise ticket prices for this year's talent show. The council has determined that for every $1 increase in price, the attendance would decrease by 20 people. What ticket price will maximize the revenue from the talent show?

2. Sasha wants to build a walkway of uniform width around a rectangular flower bed that measures 20m x 30m. Her budget is $6000 and it will cost her $10/m² to construct the path. How wide will the walkway be?

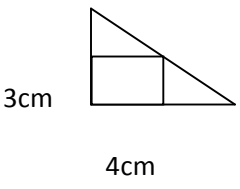
3. If you plant 100 pear trees in an acre, then the annual revenue is $90 per tree. If more trees are planted, they generate fewer pears per tree and the annual revenue per tree is decreased by $0.70 for each additional tree planted. Additionally, it costs $7.40 per tree per year for maintaining each tree. How many pear trees should be planted to maximize profit?

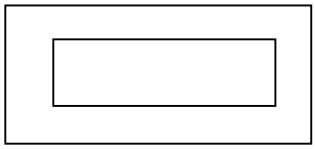
    a. What is the equation for revenue?

    b. What is the equation for profit?

    c. Find the max value for the profit equation

4. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs as shown in the figure.





5. A frame for a picture is 2½ cm wide. The picture enclosed inside the frame is 5 cm longer than it is high. If the area of the ENTIRE picture (picture & frame) is 300 cm^2, what are the dimensions of the outer frame? (see diagram above)

6. A farmer with 400 meters of fencing material wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that he can enclose? What will the dimensions be?

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1. If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning

2. If $y = -4x^2 + kx - 1$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning

3. Find the range of the parabola $y = -2(x - 4)(x + R)$

4. Find the minimum point of $y = x^2 - bx + 4$

5. Determine the value of *W* such that $f(x) = Wx^2 + 2x - 5$ has one real root. Verify your solution (i) graphically and (ii) using an alternative algebraic method.

6. Determine the value of *b* such that $f(x) = 2x^2 + bx - 8$ has no solutions. Explain the significance of your results.

7. Determine the value of *b* such that $f(x) = 2x^2 + bx + 8$ has no solutions.

8. Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real roots.

9. Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real rational roots.

10. Solve the system for *m* such that there exists only one unique solution

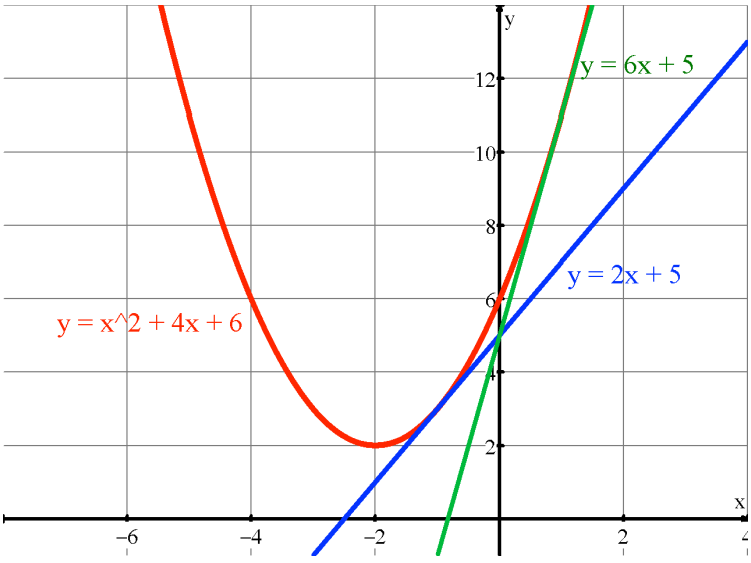
$$y = x^2 + 4x + 6$$
$$y = mx + 5$$

    a. The line(s) y = mx + 5 are called tangent lines ➔ WHY?

    b. Determine the average rate of change (slope of the line segment) between $x_1 = 2$ and $x_2 = 3$.

    c. Now, determine the average rate of change on the parabola (slope of the line segment) between $x_1 = a$ and $x_2 = a + 0.001$ where (a,b) represents the intersection point of the line and the parabola

    d. Compare this value to *m*. What do you notice?



(d) MORE QUESTIONS AT http://www.mit.edu/~alexrem/MC_Algebra2.pdf