

(A) Lesson Context

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> • How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic equations? 		
CONTEXT of this LESSON:	<p>Where we've been</p> <p>In Lessons 4,5 & 6, you learned about factoring quadratic expressions</p>	<p>Where we are</p> <p>We can algebraically solve quad eqns by factoring <i>IF</i> the eqn is written in the form of $f(x) = 0$</p>	<p>Where we are heading</p> <p>How can I use graphs and equations to make predictions from quadratic data sets & quadratic models and quadratic equations</p>

(B) Lesson Objectives:

- Review the algebraic skills of factoring
- Understand what solving a quadratic equation in the form of $ax^2 + bx + c = 0$ means in terms of graphs
- Use the skills of factoring to solve quadratic equations

(C) FAST FIVE: Understanding Terms → SOLVE EACH EQN ALGEBRAICALLY AND THEN VERIFY GRAPHICALLY

To SOLVE an equation means:

(a) If $f(x) = 2x + 8$, solve $f(x) = 0$

(b) If $f(x) = 2x + 8$, solve $f(x) = -6$

(c) If $f(x) = x - 3$, solve $f(x) = 0$

(d) If $f(x) = x - 3$, solve $f(x) = 6$

(e) If $f(x) = 2^x - 8$, solve $f(x) = 0$

(f) If $f(x) = 2^x - 8$, solve $f(x) = 24$

(g) If $f(x) = x^2 - 2x - 24$, solve $f(x) = 0$

(h) If $f(x) = x^2 - 2x - 24$, solve $f(x) = -21$

(i) If $f(x) = (2x + 5)(x - 3)$, solve $f(x) = 0$

(j) If $f(x) = (2x + 5)(x - 3)$, solve $f(x) = 13$

(C) Practice with Solving by Factoring

More Quadratic Equations – Solve by Factoring

Solve by factoring:

- | | | | |
|--------------------------------------|-------------------------------------|--------------------------|-----------|
| 1. $x^2 - 2x - 24 = 0$ | 2. $t^2 - 3t - 18 = 0$ | 3. $z^2 - 6z + 5 = 0$ | 1. _____ |
| | | | 2. _____ |
| 4. $x^2 - 7x + 6 = 0$ | 5. $x^2 + 9x + 8 = 0$ | 6. $v^2 + 10v + 9 = 0$ | 3. _____ |
| | | | 4. _____ |
| 7. $x^2 - 10x + 25 = 0$ | 8. $y^2 - 12y + 36 = 0$ | 9. $10y^2 + 5y = 0$ | 5. _____ |
| | | | 6. _____ |
| 10. $8x^2 - 12x = 0$ | 11. $x^2 - 21 = 4x$ | 12. $t^2 - 20 = t$ | 7. _____ |
| | | | 8. _____ |
| 13. $2x^2 + x = 10$ | 14. $4x^2 + 12x = -9$ | 15. $5r^2 = 12 + 11r$ | 9. _____ |
| | | | 10. _____ |
| 16. $3x^2 = x + 2$ | 17. $5y^2 = 11y - 2$ | 18. $16v^2 - 8v + 1 = 0$ | 11. _____ |
| | | | 12. _____ |
| 19. $25y^2 - 10y + 1 = 0$ | 20. $x^2 - 1 = 0$ | 21. $t^2 - 25 = 0$ | 13. _____ |
| | | | 14. _____ |
| 22. $16y^2 - 1 = 0$ | 23. $4z^2 - 25 = 0$ | 24. $x + 28 = x(x - 2)$ | 15. _____ |
| | | | 16. _____ |
| 25. $y + 12 = y(y - 3)$ | 26. $x^2 - 4x - 4 = 3x^2 - 5x - 3$ | | 17. _____ |
| | | | 18. _____ |
| 27. $x^2 + 2x + 3 = (2x - 1)(x + 5)$ | 29. $x^2 + x - 1 = (2x + 1)(x + 2)$ | | 19. _____ |
| | | | 20. _____ |
| | | | 21. _____ |
| | | | 22. _____ |
| | | | 23. _____ |
| | | | 24. _____ |
| | | | 25. _____ |
| | | | 26. _____ |
| | | | 27. _____ |
| | | | 28. _____ |

(D) Changing from Factored Form to Standard Form

You are now given pairs of zeroes/x-intercepts OR you are given solutions to the equation $f(x) = 0 \rightarrow$ you must write an equation of the parabola that has these zeroes/solutions, both in factored form and in standard form

<p>SKILLS REVIEW: A fcn has two zeroes at $x = -3$ and $x = 5$ and let the value of a be 2</p>	<p>SKILLS REVIEW: A fcn has 2 zeroes at $x = 4$ and $x = 9$ and the y-intercept is $(0, -72)$</p>	<p>The fcn $y = h(x)$ has $h(-1) = h(11) = 0$ and the minimum value is -72.</p>
<p>The equation $f(x) = 0$ has solutions of $x = -3$ and $x = 2.5$ and we also know that $f(0) = 30$</p>	<p>The equation $g(x) = 0$ has solutions of $x = -3$ and $x = -3$ and we also know that $g(-5) = -8$</p>	<p>The zeroes of $y = f(x)$ are at 5 and -5. The maximum value of $f(x)$ is $\frac{25}{4}$.</p>
<p>The two solutions to the eqn $f(x) = 0$ are $x_1 = \frac{2}{3}$ and $x_2 = -\frac{1}{2}$ and we also know that $f(0) = -4$</p>	<p>The two solutions to the eqn $g(x) = 0$ are $x_1 = \frac{5}{7}$ and $x_2 = -\frac{4}{3}$ and we also know that $g(0) = 5$.</p>	<p>The two solutions to the eqn $h(t) = 0$ are $t_1 = -0.05$ and $t_2 = 0.20$ and we also know that $h(0) = -0.1$.</p>

(E) Solving (by Factoring) Quadratic Equations → GDC-I Application Problems

11. A model rocket is shot into the air and its path is approximated by $h = -5t^2 + 30t$, where h is the height of the rocket above the ground in metres and t is the elapsed time in seconds.
- When will the rocket hit the ground?
 - What is the maximum height of the rocket?
12. A baseball is thrown from the top of a building and falls to the ground below. Its path is approximated by the relation $h = -5t^2 + 5t + 30$, where h is the height above ground in metres and t is the elapsed time in seconds.
- How tall is the building?
 - When will the ball hit the ground?
 - When does the ball reach its maximum height?
 - How high above the building is the ball at its maximum height?
13. **Application:** A small company that manufactures snowboards uses the relation $P = 162x - 81x^2$ to model its profit. In the model, x represents the number of snowboards in thousands, and P represents the profit in thousands of dollars.
- What is the maximum profit the company can earn?
 - How many snowboards must it produce to earn this profit?
 - The company breaks even when there is neither a profit nor a loss. What are the break-even points for the company?
14. A computer software company models the profit on its latest game using the relation $P = -2x^2 + 28x - 90$, where x is the number of games it produces in hundred thousands and P is the profit in millions of dollars.
- What is the maximum profit the company can earn?
 - How many games must it produce to earn this profit?
 - What are the break-even points for the company?
18. **Thinking, Inquiry, Problem Solving:** Soundz Inc. makes CD players. Last year, accountants modelled the company's profit by $P = -5x^2 + 60x - 135$. Over the course of the year, in an effort to become more efficient, Soundz Inc. restructured its operation, eliminating some employees and reducing costs. This year, accountants are using $P = -7x^2 + 70x - 63$ to project the company's profit. In both models, P is the profit in hundreds of thousands of dollars and x is the number of CD players made, in hundreds of thousands. Was Soundz Inc.'s restructuring effective? Justify your answer.

(F) Practice – Graphing & Word Problem Context (GDC-Inactive)

Apply to Problems → Mr. S. can sell 500 apples per week when he charges 50 cents per apple. Through market research, his wife (being smarter than Mr. S of course) knows that for every price increase of 2 cents per apple, he will sell 10 less apples.

- i. Determine an equation that can you used to model Mr. S.'s expected revenues.
- ii. What price should he charge to maximize his revenues?
- iii. What is his maximum revenue?
- iv. How many price increments are required such that his business has NO revenue?

Apply to Problems → The profits of a company in its first 13 months of operations are modelled by the quadratic function $P(m) = -0.25m^2 + 3m - 5$ where m is the number of months (and $m = 0$ represents January 1st and $m = 1.5$ represents mid-February) and $P(m)$ is measured in billions of pesos.

- a. Determine when the company “breaks even”.
- b. Determine in which month the company maximizes its profits.
- c. What are the company’s maximum profits?
- d. Solve and interpret $P(m) < 0$ given that the domain is
- e. For what values of m are the profits DECREASING? Explain how you determined your answer.
- f. Solve $P(m) = -12$ and interpret