(A) Lesson Context

BIG PICTURE of this UNIT:	• How can I analyze growth or decay patterns in data sets & contextual problems? • How can I algebraically & graphically summarize growth or decay patterns? • How can I compare & contrast linear and exponential models for growth and decay problems.		
CONTEXT of this LESSON:	Where we've been In Lessons 3 & 4, you looked at how exponential equations can be used to model real world scenarios	Where we are How can we solve exponential equations that arise when we model growth & decay patterns	Where we are heading How can I use algebra, data tables, graphs & equations to make predictions about scenarios which feature exponential growth & decay?

(A) Lesson Objectives

a. Use algebraic strategies to solve Exponential equations
b. Use multiple representations to verify algebraic solutions
c. Apply Exponential Equations to real world applications

(B) Intro to Algebraic Strategies - Solve for the variable:

1. $y = 3^3$

2. $27 = x \cdot 3^2$

3. $27 = x^3$

4. $27 = 3^x$

5. $y = 17^4$

6. $500 = x \cdot 4^4$

7. $76 = x^9$

8. $56 = 5^x$

(C) Exponential Equations – Solving algebraically in context: $y = a(1 + r)^t$

Solving for y:

1. You deposit $1600 in a bank account. Find the balance after 3 years if the account pays 2.5% annual interest compounded quarterly.

2. The population of HS students at CAC can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11% per year. If there were 350 students present in 2013, how many HS students would be predicted to be at CAC in 2020?

Solving for a:

3. In 8 years, you want the money you invest to reach $10,000. The account pays 8% annual interest compound monthly. How much money do you need to invest?

4. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11% per year. There are currently 320 HS students at CAC. How many where present in 2000?

Solving for t:

5. You buy a new computer for $2100. The computer decreases in value by 50% annually. When will the computer be worth $600?

6. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. How long would it take for the student population to decline from 350 students to 250 students?

Solving for r:

7. After investing $2000 for 15 years, you now have $8,000. What interest rate does the investment pay annually?

8. The population of HS students at CAC can be modeled with an exponential function. If there were 370 students present in 2011 and 315 students in 2014, what is the annual rate of decrease of student population in HS at CAC?

(D) Exponential Equations – Opening Example: Investigation #1

    a. Use ALGEBRAIC METHODS to solve and verify these equations. Finally, use your TI-84 to graphically verify.

(a) Solve and verify $2^{3-x} = 2^4$	(b) Solve and verify $2^{x-3} = 2^{3x+1}$
(c) Solve and verify $2^{2x+3} = 16$	(d) Solve and verify $8^x = 16^{x-1}$

(E) Examples of Equations to Solve

$2^{1-2x} = 8$	$4^{1+x} = 2$
$3^{x+2} = \dfrac{1}{9}$	$4^{2-x} = 5$
$2^{3-2x} = 2^x$	$4^{x-1} = 2^x$
$\left(\dfrac{1}{4}\right)^{2x+1} = \left(\dfrac{1}{8}\right)^{3-x}$	$3^{2x-2} = 2^x$

(F) Exponential – Constant Systems

EXPLORATORY EXAMPLE #1 ➔ **Solve the equation** $2^{1-2x} = 8$. **Verify your solution.**

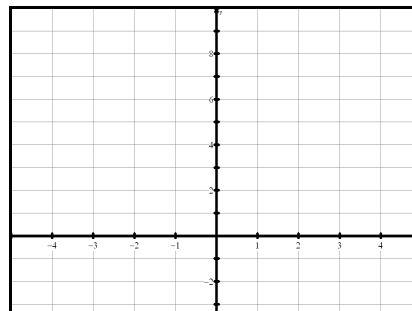**Algebraic Solution**   **Verification:**   **Graphic Solution (from TI-84)**

KEY CONCEPT ➔   Let $y_1 =$ and let $y_2 =$

EXPLORATORY EXAMPLE #2 ➔ **Solve the equation** $4^{1+x} = 2$. **Verify your solution.**

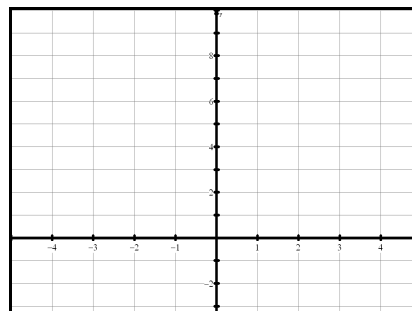**Algebraic Solution**   **Verification:**   **Graphic Solution (from TI-84)**

KEY CONCEPT ➔   Let $y_1 =$ and let $y_2 =$

(G)Exponential – Constant Systems

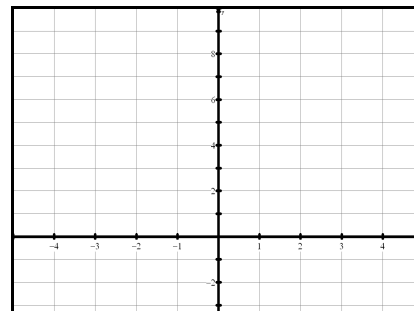**EXPLORATORY EXAMPLE #3 ➔ Solve the equation** $3^{x+2} = \dfrac{1}{9}$**. Verify your solution.**

Algebraic Solution

KEY CONCEPT ➔

Verification:

Graphic Solution (from TI-84)

Let $y_1 =$ and let $y_2 =$

EXPLORATORY EXAMPLE #4 ➔ Solve the equation $4^{2-x} = 5$**. Verify your solution.**

Algebraic Solution

KEY CONCEPT ➔

Verification:

Graphic Solution (from TI-84)

Let $y_1 =$ and let $y_2 =$

(H)Exponential – Exponential Systems

EXPLORATORY EXAMPLE #1 ➔ **Solve the equation** $2^{3-2x} = 2^x$ **. Verify your solution.**

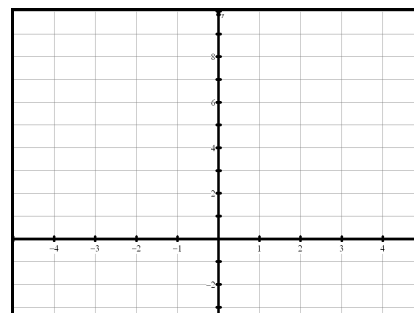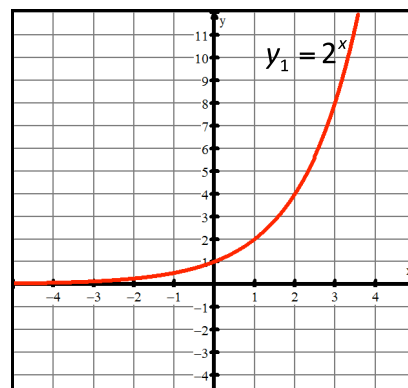**Algebraic Solution**   **Verification:**   **Graphic Solution (from TI-84)**

KEY CONCEPT ➔   Let $y_1 =$ and let $y_2 =$

EXPLORATORY EXAMPLE #1 ➔ **Solve the equation** $4^{x-1} = 2^x$ **. Verify your solution.**

Algebraic Solution   **Verification:**   **Graphic Solution (from TI-84)**
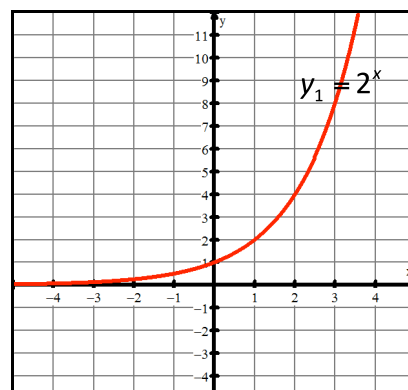
KEY CONCEPT ➔   Let $y_1 =$ and let $y_2 =$

(I) Exponential – Exponential Systems

EXPLORATORY EXAMPLE #1 ➔ Solve the equation $\left(\dfrac{1}{4}\right)^{2x+1} = \left(\dfrac{1}{8}\right)^{3-x}$ **. Verify your solution.**

Algebraic Solution   **Verification:**   **Graphic Solution (from TI-84)**
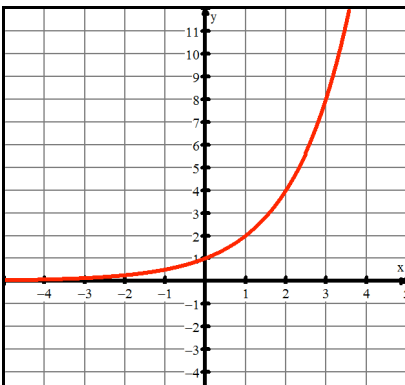
KEY CONCEPT ➔   Let $y_1 =$ and let $y_2 =$

EXPLORATORY EXAMPLE #3 ➔ Solve the equation $3^{2x-2} = 2^x$ **. Verify your solution.**

Graphic Solution   **Algebraic Solution**

Let $y_1 =$ and let $y_2 =$ KEY CONCEPT ➔

Verification:

(J) **Closing Investigation**

My brother works as an electrician and runs his own company. In the first year of running his business, he earned total revenues of $250,000 and he now estimates that his annual revenue has been increasing at a rate of 30% of the previous year's revenues. He also realizes that his business has expenses, which he estimated at $100,000 for his first year of running his business. However his expenses have been increasing at a constant, fixed amount of $55,000 every year. You will analyze the profitability of his business using appropriate mathematical modeling.

i. Write an equation for his company's REVENUES. Graph this equation on your TI-84. (Window settings x ➔ 0-25 and y ➔ 0 – 1,000,000)

ii. Write an equation for his company's EXPENSES. Graph this equation on the same axes as (i).

iii. If you know a company's revenues and expenses, how do you determine its PROFITS?

iv. Write an equation that will model the company's PROFITS.

v. What is the company's profitability in the fifth year of operation?

vi. What is the company's profitability in the 7th year of operation?

vii. What do the intersection points represent?

viii. What ASSUMPTION are you making as you analyze my brother's company's profitability?

Classwork

1. Diego decided to invest his $500 tax refund rather than spending it. He found a bank that would pay him 4% interest, compounded quarterly. If he deposits the entire $500 and does not deposit or withdraw any other amount, how long will it take him to double his money in the account?

2. William wants to have a total of $4000 in two years so that he can put a hot tub on his deck. He finds an account that pays 5% interest compounded monthly. How much should William put into this account so that he'll have $4000 at the end of two years?

3. The annual consumption of pork per person was about 35 lb in 1997 and about 20 lb in 2007. Assuming consumption is decreasing according to the exponential-decay model:
   a. Find the value of r, the rate of growth. Write the corresponding exponential equation.
   b. Estimate the consumption of pork in 2010.
   c. In what year (theoretically) will the consumption of pork be 10 lb per person?

4. Kelly plans to put her graduation money into an account and leave it there for 4 years while she goes to college. She receives $750 in graduation money that she puts it into an account that earns 4.25% interest compounded semi-annually. How much will be in Kelly's account at the end of four years?

5. ABC Bank is offering to double your money! They say that if you invest with them at 6% interest compounded quarterly they will double your money. If you invest $1500 in the account, how long will it take to double your money.

6. At what rate, converted semiannually, will $600 amount to $900 in 8 years?

7. The number of wolves in the wild in the northern section of the Cataragas county is decreasing at the rate of 3.5% per year. Your environmental studies class has counted 80 wolves in the area. After how many years will this population of 80 wolves drop below 15 wolves if this rate of decrease continues?

8. Jane bought a Saturn Vue in 2002 for $20,000. In 2007, the residual value of her Vue was $15,000.
   a. Find the value of r, the rate of growth. Write the corresponding exponential equation.
   b. What is the residual value in 2010?
   c. In what year (theoretically) will the residual value of the Vue be half of what Jane paid for it?