A. **Lesson Context**

BIG PICTURE of this UNIT:	• How can I analyze growth or decay patterns in data sets & contextual problems? • How can I algebraically & graphically summarize growth or decay patterns? • How can I compare & contrast linear and exponential models for growth and decay problems.

CONTEXT of this LESSON:	Where we've been In Lesson 3, you worked with exponential growth and decay situations	Where we are How do we work with equations that model compound interest situations	Where we are heading How can I use equations that will help me make predictions about scenarios which feature exponential growth & decay?

(A) **Lesson Objectives:**

a. Write exponential equations to model real world applications that are specific to compound interest
b. Make predictions/extrapolations through numeric or algebraic analysis
c. Use multiple representations to solve the exponential equations that arise from real world applications

(B) **Fast Five: Comparison of Compound vs Simple Interest**

Simple Interest: CASE #1. I invest $1,000 earning 10% p.a simple interest (i.e. I get 10% of $1000 added to my investment every year). I wish to monitor the value of my investment every year for the next ten years.

(a) Prepare a Data Table for CASE #1:

Compound Interest: CASE #2. I invest $1,000 earning 10% p.a. compounded annually (i.e. every year, I get 10% of the previous year's total value added to my investment). I wish to monitor the value of my investment every year for the next ten years.

(a) Prepare a Data Table for CASE #2:

(b) Use your TI-84 to graph each investment option and compare and contrast the two investment options. Sketch the graphs

(c) Calculate the value of each investment at the end of a 20 year time period.

(C) COMPOUND INTEREST: Example to Investigate
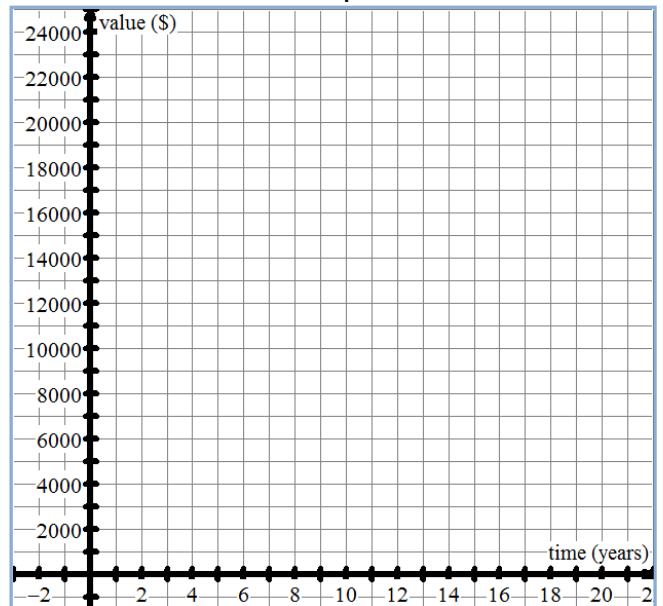
a. When my oldest son, Alexander, was born, I invested $5,000 in an education fund for him. The education fund is earning 8% **interest compounded every year**.
    i.   How much this investment is worth when Alexander starts university at the age of 19 years old?
    ii. When has the investment tripled its value?

Data table		Eqn	Graph

X (time in years)	Y (value in $)

value ($)
24000
22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
time (years)
-2   2   4   6   8   10   12   14   16   18   20   2

(a) Value when Alex is 19 ➔ (write eqn)

(b) When has the investment tripled in value ➔ (write eqn)

(c) What assumption are you making ➔

(D) Compound Interest Formula

b. When the interest is paid out at the END OF EVERY YEAR, we say that the compounding conditions are "compounded annually" at *x*%/a.

c. In this case, our general formula for exponential equations is $y = a(b)^x$ will become "modified/rewritten/represented" as $FV = PV(1 + r)^t$. The PARAMETERS (FV, PV, r, t) mean:

FV ➔	PV ➔	r ➔	t ➔

(E) Examples

a. Martina invests $5000 in a savings account that pays 5.25%/a, compounded annually. She does not make another deposit.
    i.   Determine the amount in the account after 20 years.
    ii. Determine when the account has a balance of $12,500.
    iii. Determine the **total interest** earned at this time

b. Natalie invests $10 000 at 8%/a, compounded annually when she is 20 years old. Allison invests $10,000 at 8% p.a. compounded annually when she is 45 years old. Both women "cash in" their investment when they turn 65 years old..
    iv. Determine the value of each woman's investment when they turn 65.
    v. Determine the **total interest** earned by each woman at this time.

c. Determine the present value of an investment that will be worth $5000 in ten years. The interest rate is 4.8%/a, compounded annually.

(F) *Examining Changes in the Compounding Conditions*

When interest is "paid" to the investor, it DOES NOT HAVE TO BE ANNUALLY!!!. What if an investor (like me) wants the interest paid MORE FREQUENTLY? How does this change the value of an investment?? How does it change the formula that I can use to predict future values?

Let's go back to my first example: When my oldest son, Alexander, was born, my wife and I invested $5,000 in an education fund for him. The education fund is earning 8% interest every year. You will develop an answer to my original questions

   a. How much this investment is worth when Alexander starts university at the age of 19 years old?
   b. When has the investment tripled its value?

Now I will have 4 investment options that you will investigate:

OPTION A ➔ 8%/a compounded semi-annually   OPTION B ➔ 8%/a compounded quarterly

OPTION C ➔ 8%/a compounded monthly   OPTION D ➔ 8%/a compounded daily

(G)*Summary*

   a. Does the value of my investment for Alex change in value given the different compounding conditions? Any ideas as to WHY/WHY NOT?

   b. Does the time taken to triple my investment change given the different compounding conditions? Any ideas as to WHY/WHY NOT?

   c. Does the formula I use to predict future values change given the different compounding conditions?

(H) Examples ➔ For each situation, determine: (i) the amount (value of the investment)   (ii) the interest earned

(a) $4000 borrowed for 4 years at 3%/a, **compounded annually**

(b) $7500 invested for 6 years at 6%/a, **compounded monthly**

(c) $15 000 borrowed for 5 years at 2.4%/a, **compounded quarterly**

(d) $28 200 invested for 10 years at 5.5%/a, **compounded semi-annually**

(e) $850 financed for 1 year at 3.65%/a, **compounded daily**

(f) $2225 invested for 47 weeks at 5.2%/a, **compounded weekly**

(g) Determine the amount that I must invest today at 6% p.a. compounded monthly, if I wish to earn $25,000 in 8 years.

(h) Determine the amount that I must invest today at 4.5% p.a. compounded daily, if I wish to earn $35,000 in 12 years.

(I) Internet Resources

      a. Video #1: from PatrickJMT ➔ http://www.youtube.com/watch?v=B3IdfBcXrLA

      b. Video #2: from PatrickJMT ➔ http://www.youtube.com/watch?v=3vN-6DA79N0

      c. Reading & Examples From PurpleMath ➔ http://www.purplemath.com/modules/expofcns4.htm

(J) Homework

From the Nelson 11 Textbook, Chap 1.8, p 70, Q7,8,9,10,11

(http://mrsantowski.tripod.com/2011IntegratedMath2/HW/M11SB061.pdf)

From the following worksheet, do 2 or 3 from each section

(http://mrsantowski.tripod.com/2014IntegratedMath2/Homework/IM2L4.4_Compound_Interest_DAY_2.pdf)