A. Lesson Context

BIG PICTURE of this UNIT:	• How can I analyze growth or decay patterns in data sets & contextual problems? • How can I algebraically & graphically summarize growth or decay patterns? • How can I compare & contrast linear and exponential models for growth and decay problems.		
CONTEXT of this LESSON:	Where we've been In Lessons 1,2 you generated & analyzed data from a variety of activities	Where we are How do we work with equations that model growth & decay patterns	Where we are heading How can I use equations that will help me make predictions about scenarios which feature exponential growth & decay?

(A) Lesson Objectives:

a. Write exponential equations to model real world applications that are specific to compound interest
b. Make predictions/extrapolations through numeric or algebraic analysis
c. Use multiple representations to solve the exponential equations that arise from real world applications

(B) Opening Exercises

Which of the exponential functions below show **growth** and which show **decay**?

a) $y = 5(2)^x$ b) $y = 100(.5)^x$ c) $y = 80(1.3)^x$

d) $y = 20(0.8)^x$ e) $y = 20(1+0.025)^x$ f) $y = 40(1-0.4)^x$

Working with the equation $y = 80(1.3)^x$,:

(a) Evaluate for y when x = 5

(b) SOLVE for x when y = 275

1. Since January 1980, the population of the city of Brownville has grown according to the mathematical model $y = 720{,}500(1.022)^x$, where x is the number of years since January 1980.

   a. Explain what the numbers 720,500 and 1.022 represent in this model.
   b. What is the annual growth rate for this population?
   c. What would the population be in 2000 if the growth continues at the same rate.
   d. Use this model to predict about when the population of Brownville will first reach 1,000,000.

2. A population of 800 beetles is growing each month at a rate of 5%.

   a. Write an equation that expresses the number of beetles at time x.
   b. About how many beetles will there be in 8 months?

3. Your new computer cost $1500 but it depreciates in value by about 18% each year.

   a. Write an equation that would indicate the value of the computer at x years.
   b. How much will your computer be worth in 6 years?
   c. About how long will it take before your computer is worth close to zero dollars, according to your equation?

4. From 1990 to 1997, the number of cell phone subscribers S (in thousands) in the US can be modeled by the equation $S = 5535.33(1.413)^t$ where t is number of years since 1990

   a. Identify the growth factor and annual percent increase
   b. In what year was the number of cell phone subscribers about 31 million?
   c. According to the model, in what year will the number of cell phone subscribers exceed 90 million?
   d. Estimate the number of subscribers in 2010.
   e. Do you think this model can be used to predict future number of cell phone subscribers? Explain.

5. From 1991 to 1995, the number of computers C per 100 people worldwide can be modeled by $C = 25.2(1.15)^t$ where t is the number of years since 1991

   a. Identify the initial amount, the growth factor and the annual percent increase.
   b. Estimate the number of computers in 2000.
   c. In what year will the number of computer users FIRST EXCEED 30 computers per 100 people?

6. Ten grams of Carbon 14 is stored in a container. The amount C (in grams) of Carbon 14 present after t years can be modeled by $C = 10(0.99987)^t$.

   a. How much is present after 1000 years?
   b. How long does it take to reduce the amount of carbon-14 to HALF its original amount?

1. A dish has 212 bacteria in it. The population of bacteria will double every 2 days. How many bacteria will be present in . . .
   a) 8 days b) 11 days c) 4 hours d) 2 months

2. An experiment starts off with *X* bacteria. This population of bacteria will double every 7 days and grows to 11,888 in 32 days. How many bacteria where present at the start of the experiment?

3. A bacteria culture grows according to the formula: $y = 12000(2)^{\frac{t}{4}}$ where t is in hours. How many bacteria are present:
   (a) at the beginning of the experiment?
   (b) after 12 hours?
   (c) after 19 days?
   (d) What is the doubling time of the bacteria?

4. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48 000 bacteria present. What is the length of the doubling period?

5. Mr S. makes an initial investment of $15,000. This initial investment will double every 9 years. What is the value of this investment in . . .
   a) 20 years b) 6 years c) BLACK LEVEL: What is the YEARLY rate of increase of this investment?

6. Iodine-131 is a radioactive isotope of iodine that has a half-life of 8 days. A science lab initially has 200 grams of iodine-131. How much iodine-131 will be present in . . .
   a) 8 days b) 20 days c) 1 year d) 2 months

7. A chemical decays according to the formula: $y = 12000\left(\frac{1}{2}\right)^{\frac{t}{25}}$ where t is in time in hours and y is amount of chemical left, measured in grams. What amount of chemical is present:
   (a) at the beginning of the experiment?
   (b) after 100 hours?
   (c) after 19 days?
   (d) What is the half-life of the chemical?

8. A block of dry ice is losing its mass at a rate of 12.5% per hour. At 1 PM it weighed 50 pounds. What was its weight at 5 PM? What was the approximate half-life of the block of dry ice under these conditions?

9. BLACK LEVEL: A medical experiment starts off with *X* grams of a radioactive chemical called Mathematicus. This chemical will decay in half every 15 seconds and in the course of the experiment, will decay to 9.765 g in 2 minutes. How much Mathematicus was present at the start of the experiment?

BLACK LEVEL CHALLENGES:

QUESTION 1 - *DUE DATE:*

Mr Smith's wife has just learned that she is pregnant! Mr. Smith wants to know when his new baby will arrive and decides to do some research. On the Internet, he finds the following article:

> **Hormone Levels for Pregnant Women**
>
> When a woman becomes pregnant, the hormone HCG (human chlorionic gonadotropin) is produced in order to enable the baby to develop.
>
> During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. Commonly, the HCG levels are measured two days apart to look for this rate of growth.
>
> A woman who is not pregnant will often have an HCG level of between 0 and 5 mIU (milli-international units) per ml (milliliter).

Then Smith remembered that his wife was tested for HCG during her last two doctor visits.

1. On March 21, her HCG level was 200 mIU/ml, while two days later, her HCG level was 392 mIU/ml.. Assuming that the model for HCG levels is of the form $y = ab^x$, what equation models the growth of HCG for his wife's pregnancy?

2. Assume her HCG level was 5 mIU on the day of implantation. How many days after implantation was his wife's first doctor visit? March 21? What day did the baby most likely become implanted?

3. Smith also learned that a baby is born approximately 37 weeks after implantation. What day can Smith expect to become a father?

QUESTION 2: *Doses of Medicine*

Medicine in the body decays in an exponential way. Mr. Smith is taking some medication. On Monday Mr. Smith took the pills. On Tuesday he had 15 mg of medicine left in his body. On Friday he had 6.328125 mg left in his body.

1. Create an exponential equation modeling this situation. (Remember… starting should be on Monday)

2. When the amount of medicine in Mr. Smith's body drops below 4 mg, he needs to take another pill. When does Mr. Smith need to take more medicine?

Answer the following questions that deal with the doubling concept. Recall that the formula $y = ab^x$ which can now be rewritten as $y = C(2)^{\frac{t}{D}}$. In these two formulas, recall what the variables really mean:

$y = ab^x$	$y = C(2)^{\frac{t}{D}}$
y ➔ a ➔ b ➔ x ➔	y ➔ C ➔ t ➔ D ➔

Answer the following questions that deal with the halving concept. Recall that the formula $y = ab^x$ which can now be rewritten as $y = C\left(\frac{1}{2}\right)^{\frac{t}{H}}$. In these two formulas, recall what the variables really mean:

$y = ab^x$	$y = C\left(\frac{1}{2}\right)^{\frac{t}{H}}$
y ➔ a ➔ b ➔ x ➔	y ➔ C ➔ t ➔ H ➔