(A) Lesson Context

BIG PICTURE of this UNIT:	• How can I analyze growth or decay patterns in data sets & contextual problems? • How can I algebraically & graphically summarize growth or decay patterns? • How can I compare & contrast linear and exponential models for growth and decay problems.		
CONTEXT of this LESSON:	Where we've been In Lessons 1 & 2 you generated & analyzed data from a variety of activities	Where we are How do we work with equations & situations that model growth & decay patterns	Where we are heading How can I use equations that will help me make predictions about scenarios which feature exponential growth & decay?

(A) Lesson Objectives:

a. Write exponential equations to model real world applications using the **b** = **1 + r** connection.
b. Make predictions/extrapolations through numeric or algebraic analysis
c. Use multiple representations to solve the exponential equations that arise from real world applications

(B) Review ➔ An Exponential equation has the form $\mathbf{Y = a(b)^x}$, where **a** = initial value, **b** is the growth factor/common ratio.

For the following equations, (i) decide if they can be used to model growth or decay and (ii) determine the common ratio/growth factor at which the change happens.

$Y = 200(1.15)^x$		
$Y = 400(0.85)^x$		
$Y = 100(2)^x$		
$Y = 100(½)^x$		
$Y = 200(1.05)^x$		
$Y = 400(1.75)^x$		
$Y = 100(0.75)^x$		
$Y = 100(0.995)^x$		
$Y = 1,000(0.30)^x$		
$Y = 2500(1.5)^x$		

(C) Working with Equations & Predictable Patterns: Exponential GROWTH

a) Start with the equation $y = 200(1.15)^x$. Enter this equation into your graphing calculator.

b) Use 2nd TABLE (graph) on your TI-84 to see the data table and thus record the first 7 values on the table below: (maybe show tableset for tblstart =0 and Δtbl = 1)

X	0	1	2	3	4	5	6
Y_1							

c) Graph the function on your TI-84. Given the data table you've just looked at, what window settings seem appropriate?

d) What is the "common ratio" in this data set/equation? Explain how you determined your answer.

e) Perform the following data analysis given the data in your data table ➔ $\frac{y(1) - y(0)}{y(0)}$ and record the value on the table below (show how to do it on TI-84).

X	0	1	2	3	4	5	6
Y_1							
Calculation result:		$\frac{y(1) - y(0)}{y(0)}$	$\frac{y(2) - y(1)}{y(1)}$	$\frac{y(3) - y(2)}{y(2)}$	$\frac{y(4) - y(3)}{y(3)}$	$\frac{y(5) - y(4)}{y(4)}$	$\frac{y(6) - y(5)}{y(5)}$

f) The number you just calculated is the "percent change" How is the "percent change" value related to the common ratio?

g) Explain how you coild now "rewrite" the equation for exponential growth ($y = ab^x$) using the "percent change" concept.

(D)Working with Equations & Predictable Patterns: Exponential DECAY

h) Start with the equation $y = 200(0.80)^x$. Enter this equation into your graphing calculator.

i) Use 2nd TABLE (graph) on your TI-84 to see the data table and thus record the first 7 values on the table below: (maybe show tableset for tblstart =0 and Δtbl = 1)

X	0	1	2	3	4	5	6
Y_1							

j) Graph the function on your TI-84. Given the data table you've just looked at, what window settings seem appropriate?

k) What is the "common ratio" in this data set/equation? Explain how you determined your answer.

l) Perform the following data analysis given the data in your data table ➔ $\frac{y(1)-y(0)}{y(0)}$ and record the value on the table below (show how to do it on TI-84).

X	0	1	2	3	4	5	6
Y_1							
Calculation result:		$\frac{y(1)-y(0)}{y(0)}$	$\frac{y(2)-y(1)}{y(1)}$	$\frac{y(3)-y(2)}{y(2)}$	$\frac{y(4)-y(3)}{y(3)}$	$\frac{y(5)-y(4)}{y(4)}$	$\frac{y(6)-y(5)}{y(5)}$

m) The number you just calculated is the "percent change" How is the "percent change" value related to the common ratio?

n) Explain how you coild now "rewrite" the equation for exponential growth ($y = ab^x$) using the "percent change" concept.

(E) <u>Opening Exploration</u>➔ Mr Santowski has been given a new job contract. He will earn $40,000 per year and get a raise of 6% of his previous years' salary (i.e his salary grows by 6% per year). To begin investigating this problem:

a) Define the variables that you will be using to model this problem.

b) Write an equation for Mr. S's salary.

c) Graph the function on your TI-84

d) What does the y-intercept represent?

e) What would my salary be in 8 years?

f) After how many years would my salary be $70,000?

g) What assumption are you making as you answer Qe,f?

h) I would like Mr. S's salary to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

(F) <u>Opening Exploration</u> ➔ Mr Santowski has purchased a new car. It cost $50,000 but its value is depreciating at a rate of 12%.

a) Define the variables that you will be using to model this problem.

b) Write an equation for the value of Mr. S's car.

c) Graph the function on your TI-84.

d) What does the y-intercept represent?

e) What would be the value of my car be in 8 years?

f) After how many years would the value of my car be $7,000?

g) I would like the value of Mr. S's car to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

(G)Examples: For each question, show your equation and a sketch of your graph.

a. A colony of 1,000 ants can increase by 25% in a month.

   i. How many ants will be in the colony after 10 months?

   ii. How long will it take to get 7,500 ants in the colony?

b. A town's population of 10,000 people will decrease by 8% every year.

   i. What will be the population after 4 years?

   ii. How long will it take to get 6,500 people?

   iii. BLACK LEVEL: Determine the MONTHLY growth rate for the town.

c. A baby weighing 7 pounds at birth may increase in weight every month according to the function $W(m) = 7(1.11)^m$.

   i. How much will the baby weigh after 1 year?

   ii. When will the baby weigh 18 pounds?

   iii. BLACK LEVEL: Determine the yearly rate of growth for this infant.

   iv. BLACK LEVEL: Determine the approximate DAILY rate of growth for this infant.

d. A deposit of $1500 in an account pays interest on the balance annually and the account balance is modeled by the function $B(t) = 1500(1.0725)^t$.

   i. Determine the yearly rate of increase of the account balance.

   ii. What is the account balance after 8 years?

   iii. When will the value of the account be double its original value?

(H) **Examples :** For each question, show your equation and a sketch of your graph

a. A colony of 100,000 ants is infected by a virus and its monthly population is modeled by the following function: $P(m) = 100000(0.88)^m$.
   i. How many ants will be in the colony after 10 months?
   ii. How long will it take to get 25,000 ants in the colony?
   iii. BLACK LEVEL: Determine the YEARLY rate of decrease of ant population.
   iv. BLACK LEVEL: Determine the DAILY death rate for the ant colony.

b. An investment of $150,000 in an account loses value at a rate of 3.25% annually.
   i. What is the account balance after 5 years?
   ii. When will the value of the account be half its original value?

c. A sample of 100 g radioactive plutonium-238 has a half-life of 87.7 years, so it will exponentially decay every year.
   i. What amount will remain after 400 years?
   ii. How long will it take to eliminate 95% of the plutonium?
   iii. BLACK LEVEL: Determine the YEARLY decay rate for plutonium

(I) **Homework Links:**

http://frontenacss.limestone.on.ca/teachers/dcasey/0F7D449A-00870BC8.23/Exp%20Growth%20Decay%20Word%20Probs.pdf