(A) Lesson Context

<table>
<tr><td>BIG PICTURE of this UNIT:</td><td colspan="3">How can I analyze growth or decay patterns in data sets & contextual problems?
How can I algebraically & graphically summarize growth or decay patterns?
How can I compare & contrast linear and exponential models for growth and decay problems.</td></tr>
<tr><td>CONTEXT of this LESSON:</td><td>Where we've been

In Gr. 8, you studied exponents and graphs of exponential relations</td><td>Where we are

What patterns/relationships exist in data sets that exhibit growth & decay patterns</td><td>Where we are heading

How can I develop equations that will help me make predictions about scenarios which feature exponential growth & decay?</td></tr>
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(B) Lesson Objectives:

a. Generate data through various hands-on activities
b. Analyze the data to look for patterns in the data that was generated
c. Make predictions/extrapolations through numeric or algebraic analysis

(C) Pay It Forward

Can you think of an idea for world change, and put it into practice?

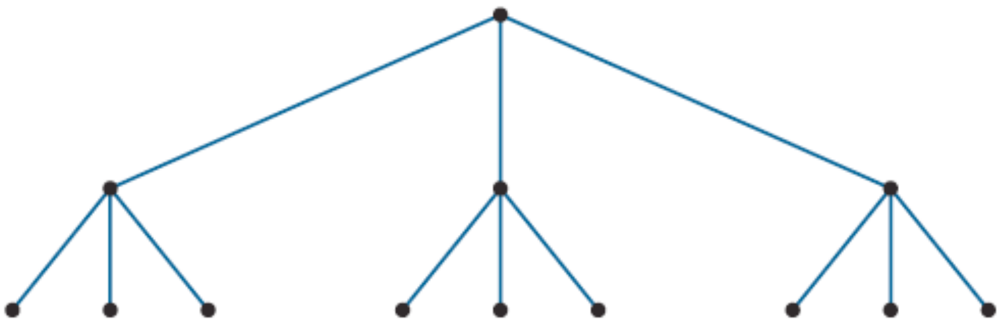
Joey came up with an idea that fascinated his mother, his teacher, and his classmates. He suggested that he would do something really good for three people. Then when they ask how they can pay him back for the good deeds, he would tell them to "pay it forward" - each doing something good for three other people.

Joey figured that those three people would do something good for a total of nine others. Those nine would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people all around the world.

In this lesson, you will find strategies for analyzing patterns of change called exponential growth. You will also discover some basic properties of exponents that allow you to write exponential expressions in useful equivalent forms.

Counting in Tree Graphs

The number of good deeds in the Pay It Forward pattern can be represented by a tree graph that starts like this:



The vertices represent the people who receive and do good deeds. Each edge represents a good deed done by one person for another.

Problem 1.

At the start of the Pay It Forward process, only one person does good deeds - for three new people. In the next stage, the three new people each do good things for three more new people. In the next stage, nine people each do good things for three more new people, and so on, with no person receiving more than one good deed.

a. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the Pay It Forward process. Then plot the (stage, number of good deeds) data.

Stage	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	3	9	27							

b. How does the number of good deeds at each stage grow as the tree progresses? How is that pattern of change shown in the plot of the data?

c. How many stages of the Pay It Forward process will be needed before a total of at least 25,000 good deeds will be done?

Problem 2.

Consider now how the number of good deeds would grow if each person touched by the Pay It Forward process were to do good deeds for only two other new people, instead of three.

a. Make a tree graph for several stages of this Pay It Forward process.

b. Make a table showing the number of good deeds done at each of the first 10 stages of the process and plot those sample (stage, number of good deeds) values.

c. How does the number of good deeds increase as the Pay It Forward process progresses in stages? How is that pattern of change shown in the plot of the data?

d. How many stages of this process will be needed before a total of 25,000 good deeds will have been done?

Problem 3.

In the two versions of Pay It Forward that you have studied, you can use the number of good deeds at one stage to calculate the number at the next stage.

a. Use the words NOW and NEXT to write rules that express the two patterns. That is, rules of the form

NEXT = …

b. How do the numbers and calculations indicated in the rules express the patterns of change in tables of (stage, number of good deeds) data?

c. Write a rule relating NOW and NEXT that could be used to model a Pay It Forward process in which each person does good deeds for four other new people. What pattern of change would you expect to see in a table of (stage, number of good deeds) data for this Pay It Forward process?

Problem 4.

It is also convenient to have rules that will give the number of good deeds N at any stage x of the Pay It Forward process, without finding all the numbers along the way to stage x.

When students in one class were given the task of finding such a rule for the process in which each person does three good deeds for others, they came up with four different ideas:

$N = 3x$

$N = x + 3$

$N = 3^x$

$N = 3x + 1$

a. Are any of these rules for predicting the number of good deeds N correct? How do you know?

b. How can you be sure that the numbers and calculations expressed in the correct "N = ..." rule will produce the same results as the NOW-NEXT rule you developed in Problem 3?

c. Write an "N = ..." rule that would show the number of good deeds at stage number x if each person in the process does good deeds for two others.

d. Write an "N = ..." rule that gives the number of good deeds at stage x if each person in the process does good deeds for four others.

Problem 5.

The patterns in spread of good deeds by the Pay It Forward process occur in other quite different situations. For example, when bacteria infect some part of your body, they often grow and split into pairs of genetically equivalent cells over and over again.

a. Suppose a single bacterium lands in a cut on your hand. It begins spreading an infection by growing and splitting into two bacteria every 20 minutes.

i. Complete a table showing the number of bacteria after each 20-minute period in the first three hours. (Assume none of the bacteria are killed by white blood cells.)

ii. Plot the (number of time periods, bacteria count) values.

iii. Describe the pattern of growth of bacteria causing the infection.

b. Use NOW and NEXT to write a rule relating the number of bacteria at one time to the number 20 minutes later. Then use the rule to find the number of bacteria after fifteen 20-minute periods.

c. Write a rule showing how the number of bacteria N can be calculated from the number of stages x in the growth and division process.

d. How are the table, graph, and symbolic rules describing bacteria growth similar to and different from the Pay It Forward examples?

e. How are they similar to, and different from, patterns of linear functions?

(D) **Number Patterns in Data Sets** - You are now given the following data sets. For EACH data set, you will:

a. write out the pattern that you observe in the data set, that you can use to make predictions about the terms that follow
b. Record the next 6 numbers in the data set
c. Write an equation (or develop an alternative plan) that will allow you predict/calculate the 25th number in your data set

Data Set #1 ➔ {3,6,12,24,48,96,192,…} ➔and as a data table ➔

x	1	2	3	4	5	6	7
y	3	6	12	24	48	96	192

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

Data Set #2 ➔ {10,20,30,40,50,60,70,….} ➔as a data table ➔

X	1	2	3	4	5	6	7
y	10	20	30	40	50	60	70

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

Data Set #3 → {80,65,50,35,20,5,-10,...} → and as a data table →

x	1	2	3	4	5	6	7
y	80	65	50	35	20	5	-10

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

Data Set #4 → {640,320,160,80,40,20,10,...} →as a data table →

X	1	2	3	4	5	6	7
y	640	320	160	80	40	20	10

Describe the pattern in words

List the next 6 numbers in the data set, given the pattern you determined

Formula/equation/method for determining the 25th number in your data set

Data Set #5: $\left\{\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, ...\right\}$ or as a data table

X	1	2	3	4	5	6	7
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

Describe the pattern in words

Formula/equation/method for determining the 25^{th} number in your data set

List the next 6 numbers in the data set, given the pattern you determined

Data Set #6

Year	1825	1850	1875	1900	1925	1950	1975
Population (in thousands)	200	252	318	401	504	635	800

Describe the pattern in words

Formula/equation/method for determining the 25^{th} number in your data set