Lesson Context

Where we've been:
You know how to find a midpoint, length of a segment, and slope of a line with and without Geogebra.

Where we are:
Use length, slope, and midpoint to classify geometric figures.

Where we are heading:
Proving geometric properties of quadrilaterals and triangles.

Lesson Objectives:

Review the properties of quadrilaterals and triangles through geogebra.
Use algebraic methods to classify quadrilaterals & triangles.

Exercise 1

Exploring Triangles

Equilateral Triangle: A(30,30); B(-41,11), C(11,-41)

State the defining property:

Confirm the property algebraically:

Isosceles triangle: A(3,-1); B(7,1); C(3,4)

State the defining property:

Confirm the propery algebraically:

Scalene triangle: A(-1,5), B(8,-2), C(-5,-1)

State the defining property:

Confirm the propery algebraically:

Right triangle: A(1,5), B(9,-5), C(-4,1)

State the defining property:

Confirm the propery algebraically:

Right isosceles triangle: A(1,5), (8,1), C(-3,-2)

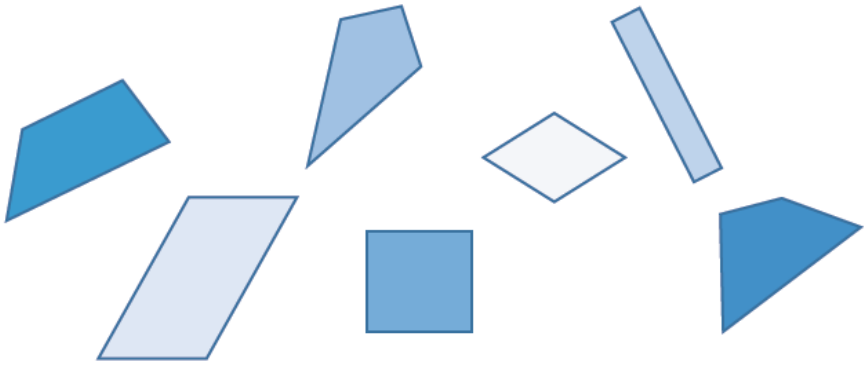
State the defining property:

Confirm the propery algebraically:

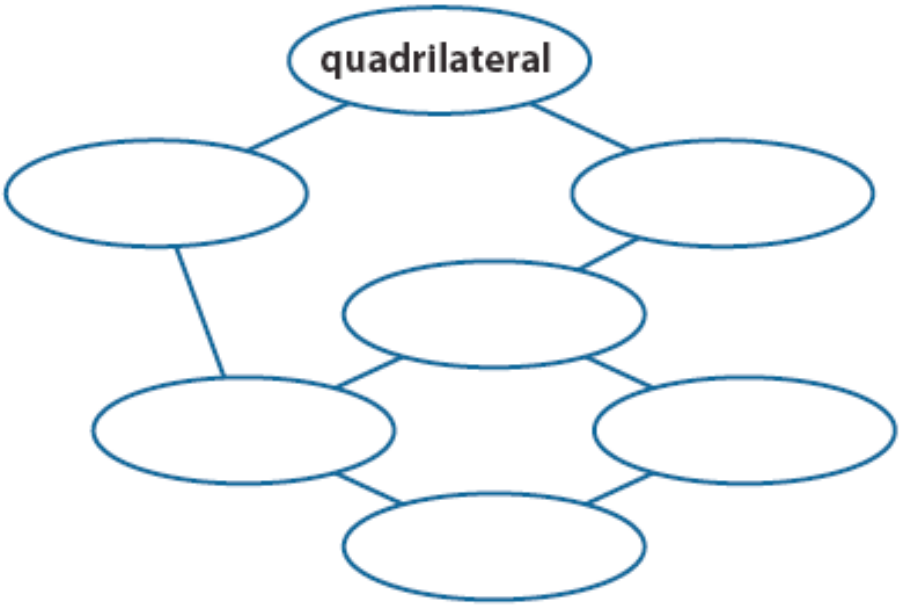
Exercise 2
Exploring Quadrilaterals

Quadrilaterals come in a variety of specialized shapes. There are ***trapezoids***, ***kites***, ***parallelograms***, ***rhombuses***, ***rectangles***, and ***squares***, as well as quadrilaterals with no special additional characteristics.

A1. **BOARD WORK**: Your group will be assigned a quadrilateral. Go online, find and list 5 "characteristics/properties" of your quadrilateral



A2. BOARD WORK: Complete a copy of the quadrilateral tree shown below by filling in each oval with the name of one of the six special quadrilaterals listed. If the oval for a quadrilateral is connected to one or more ovals above it, then that quadrilateral must also have the properties of the quadrilateral(s) in the ovals above it to which it is connected.



A**3.** Suppose you know that a characteristic X (such as opposite sides congruent) is true for parallelograms. For which other quadrilaterals must characteristic X also be true? Explain your reasoning.

A4. Suppose a characteristic Y is true for all kites. Which other quadrilaterals must also have characteristic Y?

B. *Properties of Parallelograms*

An IM2 class at CAC was asked to make a list of properties of every parallelogram, and students prepared the following list. Use GEOGEBRA to examine each statement to see if you think it is correct. If you think it is correct, construct a convincing argument to support your case. If you think it is incorrect, give a ***counterexample***. Share the work within your group but be prepared to explain your conclusions and reasoning to the entire class.

If a quadrilateral is a parallelogram, then:

a. each diagonal divides the parallelogram into two congruent triangles.

b. opposite angles are congruent.

c. consecutive angles are supplementary.

d. the diagonals are congruent.

e. the diagonals bisect each other.

f. the diagonals are perpendicular.

3. How would your answers to Problem B change if "parallelogram" was replaced by:

a."rectangle"?

b."rhombus"?

c."kite"?

Consolidating Skills: Practice the following questions and submit for grading

GREEN LEVEL:

7. $P(-7, 1)$, $Q(-8, 4)$, and $R(-1, 3)$ are the vertices of a triangle. Show that $\triangle PQR$ is a right triangle.

8. A triangle has vertices at $L(-7, 0)$, $M(2, 1)$, and $N(-3, 5)$. Verify that it is a right isosceles triangle.

11. A polygon is defined by points $R(-5, 1)$, $S(5, 3)$, $T(2, -1)$, and $U(-8, -3)$. Show that the polygon is a parallelogram.

12. A quadrilateral has vertices at $A(-2, 3)$, $B(-2, -2)$, $C(2, 1)$, and $D(2, 6)$. Show that the quadrilateral is a rhombus.

13. **a)** Show that $EFGH$, with vertices at $E(-2, 3)$, $F(2, 1)$, $G(0, -3)$, and $H(-4, -1)$, is a square.

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b) Show that the diagonals of $EFGH$ are perpendicular to each other.

16. Determine the type of quadrilateral described by each set of vertices. Give reasons for your answers.

a) $J(-5, 2)$, $K(-1, 3)$, $L(-2, -1)$, $M(-6, -2)$

b) $E(-5, -4)$, $F(-5, 1)$, $G(7, 4)$, $H(7, -1)$

c) $D(-1, 3)$, $E(6, 4)$, $F(4, -1)$, $G(-3, -2)$

d) $P(-5, 1)$, $Q(3, 3)$, $R(4, -1)$, $S(-4, -3)$

BLACK LEVEL:

18. Points $P(4, 12)$, $Q(9, 14)$, and $R(13, 4)$ are three vertices of a rectangle.

T **a)** Determine the coordinates of the fourth vertex, S.

b) Briefly describe how you found the coordinates of S.

c) Predict whether the lengths of the diagonals of rectangle $PQRS$ are the same length. Check your prediction.

Extending

20. a) Show that the midpoints of any pair of sides in a triangle are two of the vertices of another triangle, which has dimensions that are exactly one-half the dimensions of the original triangle and a side that is parallel to a side in the original triangle.

b) Show that the midpoints of the sides in any quadrilateral are the vertices of a parallelogram.

13. a) Show that points $A(-4, 3)$ and $B(3, -4)$ lie on $x^2 + y^2 = 25$.

b) Show that the perpendicular bisector of chord AB passes through the centre of the circle.

14. A trapezoid has vertices at $A(1, 2)$, $B(-2, 1)$, $C(-4, -2)$, and $D(2, 0)$.

A **a)** Show that the line segment joining the midpoints of BC and AD is parallel to both AB and DC.

b) Show that the length of this line segment is half the sum of the lengths of the parallel sides.

15. $\triangle ABC$ has vertices at $A(3, 4)$, $B(-2, 0)$, and $C(5, 0)$. Prove that the

T area of the triangle formed by joining the midpoints of $\triangle ABC$ is one-quarter the area of $\triangle ABC$.