

**(A) Lesson Context**

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>mastery with algebraic skills to be used in our work with co-ordinate geometry (midpoint, length, slope)</li> <li>understanding various geometric properties of quadrilaterals &amp; triangles</li> <li>how do you really prove that something is “true”?</li> </ul>		
CONTEXT of this LESSON:	<p>Where we’ve been</p> <p>In MS, you have been taught about various types of geometric figures like quadrilaterals &amp; triangles</p>	<p>Where we are</p> <p>Becoming proficient with one analytical tool that we can use in co-ordinate geometry → midpoint</p>	<p>Where we are heading</p> <p>How can I prove various geometric properties of quadrilaterals and triangles?</p>

**(B) Lesson Objectives:**

- a. Exploring the midpoint and length of a line segment through dynamic geometry software (geogebra)
- b. Develop proficiency in analytic/algebraic determination of midpoints of line segments and length
- c. Apply the use of midpoints/length formulas to problem solving questions

**(C) EXPLORATION ASSIGNMENT #1: Dynamic geometry software: Working with Geogebra**

1. Show me an axes with a grid	
2. Show me that you can construct a line segment between 2 points and measure its length, slope and find its midpoint	
3. Show me that you can construct a line through two points & determine the slope and equation	
4. Show me that you can construct a triangle and measure the slope of each side and the area	
5. Show me you can reflect a triangle across the x-axis and across the y-axis	
6. Show me that you can translate a triangle 3 units to the left and 6 units down	
7. Show me that you can construct a perpendicular bisector of a side of a triangle	
8. Show me that you can construct an angle bisector of an angle in a triangle	

### (D) Line Segment Recap from last class:

A line segment has an endpoint at  $A(5,2)$  and midpoint at  $M(9,-3)$ . Determine the co-ordinates of the other endpoint. Show the algebraic reasoning/work that leads to your conclusion.

On the design plan for a landscaping project, a straight path runs from  $(11,29)$  to  $(53,9)$ . A light is going to be placed halfway along the path.

- (A) Draw a diagram that shows the path.
- (B) Determine the co-ordinates of the lamp on your diagram
- (C) The one lamp is not bright enough to illuminate the pathway. So two more lamps will be placed along the path, such that each lamp is placed a quarter of the distance of the path. Determine the coordinates of the other two lamps.
- (D) CHALLENGE: Where would you place the lights, so that there were TWO lights equally spaced between the end points of  $(11,29)$  and  $(53,9)$

Challenge: A perpendicular bisector of a line segment is a second line that will (i) cut the line segment in half and (ii) be perpendicular to the original line segment. A line segment ends at the points  $C(-2,0)$  and  $D(4,-4)$ . Determine the equation for the perpendicular bisector of line segment CD. Visualize it with Geogebra... then try and complete the problem with algebra.

### **(E) Length Recap from last class**

The formula to find the length of a line segment between two points on a graph is  $l = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

- Determine the length of the line segment between  $A(1,1)$  and  $B(5,9)$
  
- Determine the length of the line segment between  $A(-1,1)$  and  $B(5,5)$

A helicopter is travelling from Town A to Town B. A grid is overlaid on the map of this region and Town A is at  $(-70,770)$  and Town B is at  $(220,490)$  & Town C is the origin.

- Draw a diagram that shows the three towns.
- Approximately how far did the helicopter travel?
- What assumption did you make about the route of the helicopter?

Triangles can be classified according to the lengths of their sides (scalene, isosceles, equilateral). A given triangle has vertices at  $A(4,5)$ ,  $B(1,2)$  &  $C(6,1)$ .

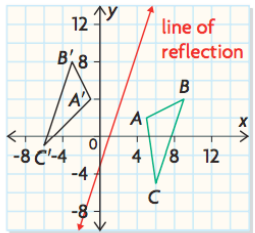
- Determine the lengths of all three sides and then classify the triangle type.
- Construct the triangle on GEOGEBRA.
- Where would you move point C such that you now had an isosceles triangle?

**CHALLENGE:** A line segment has an endpoint at  $A(5,2)$  and has a length of 13 units. Determine the co-ordinate(s) of the other endpoint. Show the algebraic reasoning/work that leads to your conclusion

# Lesson 2: Applications with Midpoint & Length | Unit 2 – Co-ordinate Geometry

Application Qs	Geometry Qs
<p>9. A forest fire is threatening two small towns, Mordon and Bently. On a map, the fire is located at <math>(10, -11)</math>, the fire hall in Mordon is located at <math>(26, 77)</math>, and the fire hall in Bently is located at <math>(12, -88)</math>. Which fire hall is closer to the fire?</p> <p>10. In a video game, three animated characters are programmed to run out of a building at <math>F(1, -1)</math> and head in three different directions. After 2 s, Animal is at <math>A(22, 18)</math>, Beast is at <math>B(-3, 35)</math>, and Creature is at <math>C(7, -29)</math>. Which character ran farthest?</p> <p>14. A coordinate grid is superimposed on the plan of a new housing development. A fibre-optic cable is being laid to link points <math>A(-18, 12)</math>, <math>B(-8, 1)</math>, <math>C(3, 4)</math>, and <math>D(15, 7)</math> in a run beginning at <math>A</math> and ending at <math>D</math>. If one unit on the grid represents 2.5 m, how much cable is required?</p> <p>15. A leash-free area for dogs is going to be created in a field behind a recreation centre. The area will be in the shape of an irregular pentagon, with vertices at <math>(2, 0)</math>, <math>(1, 6)</math>, <math>(8, 9)</math>, <math>(10, 7)</math>, and <math>(6, 0)</math>. If one unit on the plan represents 10 m, what length of fencing will be required?</p>	<p>7. A triangle has vertices at <math>A(2, -2)</math>, <math>B(-4, -4)</math>, and <math>C(0, 4)</math>.</p> <p><b>K</b> a) Draw the triangle, and determine the coordinates of the midpoints of its sides.</p> <p>b) Draw the median from vertex <math>A</math>, and determine its equation.</p> <p>9. A quadrilateral has vertices at <math>P(1, 3)</math>, <math>Q(6, 5)</math>, <math>R(8, 0)</math>, and <math>S(3, -2)</math>. Determine whether the diagonals have the same midpoint.</p> <p>11. A triangle has vertices at <math>P(7, 7)</math>, <math>Q(-3, -5)</math>, and <math>R(5, -3)</math>.</p> <p><b>A</b> a) Determine the coordinates of the midpoints of the three sides of <math>\triangle PQR</math>.</p> <p>b) Calculate the slopes of the <b>midsegments</b> of <math>\triangle PQR</math>.</p> <p>c) Calculate the slopes of the three sides of <math>\triangle PQR</math>.</p> <p>d) Compare your answers for parts b) and c). What do you notice?</p> <p>12. Determine the equations of the medians of a triangle with vertices at <math>K(2, 5)</math>, <math>L(4, -1)</math>, and <math>M(-2, -5)</math>.</p>

## Black Level/HL Extension Questions:

<p>12. Calculate the distance between each line and the point. Round your answer to one decimal place.</p> <p>a) <math>y = 4x - 2</math>, <math>(-3, 3)</math>      c) <math>2x + 3y = 6</math>, <math>(7, 6)</math></p> <p>b) <math>y = -x + 5</math>, <math>(-1, -2)</math>      d) <math>5x - 2y = 10</math>, <math>(2, 4.5)</math></p> <p>13. A new amusement park is going to be built near two major highways.</p> <p><b>T</b> On a coordinate grid of the area, with the scale 1 unit represents 1 km, the park is located at <math>P(3, 4)</math>. Highway 2 is represented by the equation <math>y = 2x + 5</math>, and Highway 10 is represented by the equation <math>y = -0.5x + 2</math>. Determine the coordinates of the exits that must be built on each highway to result in the shortest road to the park.</p>	<p>15. A triangle has vertices at <math>D(8, 7)</math>, <math>E(-4, 1)</math>, and <math>F(8, 1)</math>. Determine the coordinates of the point of intersection of the medians.</p> <p>16. In the diagram, <math>\triangle A'B'C'</math> is a reflection of <math>\triangle ABC</math>. The coordinates of all vertices are integers.</p>  <p>a) Determine the equation of the line of reflection.</p> <p>b) Determine the equations of the perpendicular bisectors of <math>AA'</math>, <math>BB'</math>, and <math>CC'</math>.</p> <p>c) Compare your answers for parts a) and b). What do you notice?</p> <p>17. A quadrilateral has vertices at <math>W(-7, -4)</math>, <math>X(-3, 1)</math>, <math>Y(4, 2)</math>, and <math>Z(-2, -7)</math>. Two lines are drawn to join the midpoints of the non-adjacent sides in the quadrilateral. Determine the coordinates of the point of intersection of these lines.</p>
<p>17. <math>\triangle ABC</math> has vertices at <math>A(1, 2)</math>, <math>B(4, 8)</math>, and <math>C(8, 4)</math>.</p> <p>a) <math>\triangle ABC</math> is translated so that vertex <math>A'</math> is on the <math>x</math>-axis and vertex <math>B'</math> is on the <math>y</math>-axis. Determine the coordinates of the translated triangle, <math>\triangle A'B'C'</math>.</p> <p>b) <math>\triangle DEF</math> has vertices at <math>D(-1, 1)</math>, <math>E(-2, 6)</math>, and <math>F(-8, 3)</math>. Is <math>\triangle DEF</math> congruent to <math>\triangle ABC</math>? Justify your answer.</p>	