1.7 Exploring Linear Systems

GOAL

Connect the number of solutions to a linear system with its equations and graphs.

YOU WILL NEED

- graphing calculator, or grid paper and ruler

EXPLORE the Math

Three different linear systems are given below.

A	B	C
$2x + 3y = -4$ $-4x - 3y = -1$	$2y = 6 - 3x$ $6x - 5 = -4y$	$x - y = 5$ $3x = 15 + 3y$

? How many solutions can a linear system have, and how can you predict the number of solutions without solving the system?

A. Solve each system of linear equations algebraically. Record the number of solutions you determine.

	A	B	C
Linear System	$2x + 3y = -4$ $-4x - 3y = -1$	$3x + 2y = 6$ $6x + 4y = 5$	$x - y = 5$ $3x - 3y = 15$
Number of Solutions			

B. Examine your algebraic solution for system A. How do you think the lines that represent the equations in this system intersect? Explain. Graph the system to check your conjecture.

C. Repeat part B for each of the other two systems.

D. For each system, explain how the graphical solution is related to the algebraic solution, and vice versa.

E. Examine the equations in each system. Are there clues that tell you how the lines will intersect? Explain.

F. Can a linear system of two equations have exactly two solutions? Can it have exactly three solutions? Explain.

G. Discuss the different cases you have identified and how each case relates to the equations and their corresponding graphs.

Reflecting

H. Both equations in a linear system that has no solution are written in the form $Ax + By = C$. Describe the relationship between the coefficients and the constants. What does this tell you about the graphs of both lines?

I. Both equations in a linear system that has an infinite number of solutions are written in the form $Ax + By = C$. Describe the relationship between the coefficients and the constants. What does this tell you about the graphs of both lines?

J. How can you tell, by looking at the equations, that a linear system has exactly one solution?

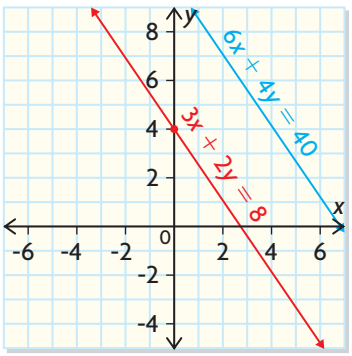
In Summary

Key Idea

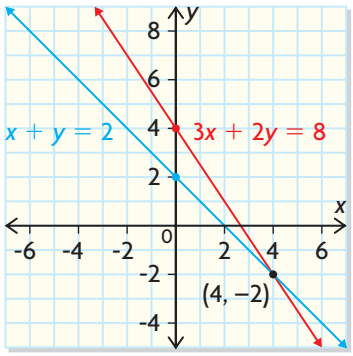
- A linear system can have no solution, one solution, or an infinite number of solutions.

Need to Know

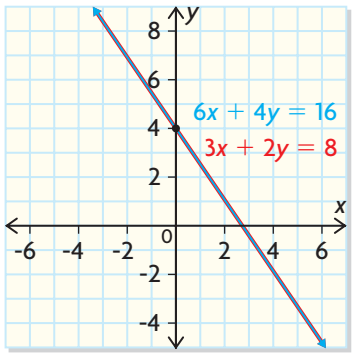
- When a linear system has no solution, the graphs of both lines are parallel and never intersect. For example, the system $3x + 2y = 8$ and $6x + 4y = 40$ does not have a solution. The coefficients in the equations are multiplied by the same amount, but the constants are not.



- When a linear system has one solution, the graphs of the two lines intersect at a single point. For example, the system $3x + 2y = 8$ and $x + y = 2$ has one solution. The coefficients and constants in the equations are not multiplied by the same amount.



- When a linear system has an infinite number of solutions, the graphs of both equations are identical and intersect at every point. For example, the system $3x + 2y = 8$ and $6x + 4y = 16$ has an infinite number of solutions. The coefficients and constants in the equations are multiplied by the same amount.



FURTHER Your Understanding

1. Graph a linear system of equations that has each number of solutions.
a) none **b)** one **c)** infinitely many

2. Use the equation $3x + 4y = 2$.
a) Write another equation that will create a linear system with each number of solutions.
i) none **ii)** one **iii)** infinitely many
b) Verify your answers for part a) algebraically and graphically.

3. Predict the number of solutions for each linear system. Then test your predictions by solving each system algebraically and verify with graphing technology.

a) $y = 3x - 5$
$y = 4x + 6$

b) $y = 4x - 3$
$y = 4x - 7$

c) $y = 5x - \frac{3}{2}$
$y = 5x - 1.5$

d) $x + 2y = 10$
$y = 8 - 0.5x$

e) $2x + 3y = 10$
$10x + 15y = 50$

f) $3x - 5y - 2 = 0$
$4x + 5y + 2 = 0$

g) $y = 1.25x - 0.375$
$5y = 4x$

h) $2x - 5 = 4y$
$0.01x - 0.02y = 0.25$

4. Create a system of linear equations that has each number of solutions. Then verify the number of solutions algebraically and graphically.
a) none **b)** one **c)** infinitely many

5. Both equations in a linear system are written in the form $Ax + By = C$. Explain how you could predict the number of solutions using the coefficients and constants of the two equations.

6. An air traffic controller is plotting the course of two jets scheduled to land in 15 min. One aircraft is following a path defined by the equation $3x - 5y = 20$ and the other by the equation $18x = 30y + 72$. Should the controller alter the paths of either aircraft? Justify your decision.