1. Here are two data tables. The first one represents the value of Mr. S's car as it is depreciating over time. I bought the car new in 2002 and the value of my car (in thousands) over the years has been tabulated below. The second data set represents world population data.

    a. For each data set, determine an exponential equation that best models the trend in the data set.

    b. Then graph the data points using the TI-84, enter your equation to determine how well your equation models the trend of the data.

<table>
<tr><td>Data Set #1</td><td>Data Set #2</td></tr>
<tr><td>
<table>
<tr><th>Year</th><th>Value</th></tr>
<tr><td>2002</td><td>40</td></tr>
<tr><td>2003</td><td>36</td></tr>
<tr><td>2004</td><td>32.4</td></tr>
<tr><td>2005</td><td>29.2</td></tr>
<tr><td>2006</td><td>26.2</td></tr>
<tr><td>2007</td><td>23.6</td></tr>
<tr><td>2008</td><td>21.3</td></tr>
<tr><td>2009</td><td>19.1</td></tr>
<tr><td>2010</td><td>17.2</td></tr>
</table>
</td><td>
<table>
<tr><th>Year</th><th>Population</th></tr>
<tr><td>1700</td><td>250</td></tr>
<tr><td>1750</td><td>370</td></tr>
<tr><td>1800</td><td>560</td></tr>
<tr><td>1850</td><td>840</td></tr>
<tr><td>1900</td><td>1270</td></tr>
<tr><td>1950</td><td>1900</td></tr>
<tr><td>2000</td><td>2850</td></tr>
</table>
</td></tr>
<tr><td>Data Analysis: Show/explain several key calculations that led to your equation:</td><td>Data Analysis: Show/explain several key calculations that led to your equation:</td></tr>
<tr><td>Equation for Data Set #1:</td><td>Equation for Data Set #2:</td></tr>
</table>

2. Exponential Functions in the Real World

The United States has continued to grow since the late 1700's. The following chart shows how the number of people per square mile changed from 1790-1980.

Year	People Per Square Mile	Year	People Per Square Mile	Year	People Per Square Mile	Year	People Per Square Mile
1790	4.5	1840	9.8	1890	17.8	1940	37.2
1800	6.1	1850	7.9	1900	21.5	1950	42.6
1810	4.3	1860	10.6	1910	26.0	1960	50.6
1820	5.5	1870	10.9	1920	29.9	1970	57.5
1830	7.4	1880	14.2	1930	34.7	1980	64

a. Graph the following data on your calculator. (Remember, enter the numbers in the list, then go back to the Menu and use Stat to graph). What is happening in the graph?

b. What is the exponential curve of best fit? Draw it as well to see if it works

c. Using the equation from Question 2, what would be the number of people per square mile in 2000? Go online and find out the actual amount of people per square mile in the United States. How close is the prediction?

Prediction: _____ Actual: _____ Difference: _____

d. Was the prediction equation a good estimator? Why or why not?

e. As stated in the title, this is an exponential function. What other type of function would create a similar outcome? Why?

3. Find a bank account balance if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

4. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

5. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

6. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

7. The population of Winnemucca, Nevada, can be modeled by $P=6191(1.04)^t$ where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

8. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

9. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

10. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left after 6 hours?

11. You deposit $1600 in a bank account. Find the balance after 3 years for each of the following situations:
    a. The account pays 2.5% annual interest compounded monthly.
    b. The account pays 1.75% annual interest compounded quarterly.

12. You buy a new computer for $2100. The computer decreases by 50% annually. When will the computer have a value of $600?

13. You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. How long until you have 10mg of caffeine?

14. The foundation of your house has about 1,200 termites. The termites grow at a rate of about 2.4% per day. How long until the number of termites doubles?

15. Solving Exponential Equations. Find all values of *x* that solve the following equations. For many problems, you may want to write both sides with the same base.

1. $2^x = 8$
2. $3 \cdot 2^x = 48$
3. $3^x = \frac{1}{9}$
4. $4^x + 7 = 71$
5. $2^x \cdot 2^{x-2} = \sqrt{2}$
6. $2^x = \frac{1}{32}$
7. $2^{x+2} = 8$
8. $\sqrt[4]{3} = 9^x$
9. $2^x = \frac{4}{\sqrt{2}}$
10. $3 \cdot 2^x = 24$
11. $2 \cdot 3^{x-3} + 1 = 19$
12. $2^x = \frac{1}{\sqrt{8}}$
13. $2 \cdot 3^{2x-1} + 7 = 61$
14. $3^x = 9^7$
15. $2^x = 2\sqrt{2}$
16. $5^x = \frac{1}{\sqrt[3]{5}}$
17. $2 \cdot 2^x = 8$
18. $\left(\frac{1}{5}\right)^x = 25$
19. $2^{2x+1} \cdot 2^x = 16$
20. $6^{-x} = \frac{6}{\sqrt[5]{6}}$
21. $\left(\sqrt{2}\right)^x = 8$
22. $3^x = \left(\frac{1}{9}\right)^{4-x}$
23. $\left(2^{x+1}\right)^2 = \frac{1}{4}$
24. $4^{2x} = 8^{x-3}$

Answers

1. x = 3
2. x = 4
3. x = -2
4. x = 3
5. x = 5/4
6. x = -5
7. x = 1
8. x = 1/8
9. x = 3/2
10. x = 3
11. x = -1
12. x = -3/2
13. x = 2
14. x = 14
15. x = 1.5
16. x = -1/3
17. x = 2
18. x = -2
19. x = 1
20. x = -4/5
21. x = 6
22. x = 8

16. Graphs of Exponential Functions - Changing the values of a and d in $y = ab^x + d$

Fill in the values in the tables, graph on graph paper & then use your TI-84 to graph the given equations and then compare them to the table and graph of the equaiton.

Equation to graph: $y = \left(\frac{1}{2}\right)^x + 1$

table:

-3	-2	-1	0	1	2	3

Equation to graph: $y = \left(\frac{1}{2}\right)^x - 3$

table:

-3	-2	-1	0	1	2	3

Equation to graph: $y = \left(\frac{1}{2}\right)^{-x} - 4$

table:

-3	-2	-1	0	1	2	3

Equation to graph: $y = 2 - \frac{1}{4}\left(\frac{1}{2}\right)^x$

table:

-3	-2	-1	0	1	2	3

Equation to graph: $y = 5 - \left(\frac{1}{2}\right)^x$

table:

-3	-2	-1	0	1	2	3

Equation to graph: $y = \frac{3}{2} + \frac{1}{2}\left(\frac{1}{2}\right)^x$

Data table:

-3	-2	-1	0	1	2	3

17. **Part I: Zero Exponents** Directions: Simplify each expression. Show all work *or* explain what you did.

1) $\frac{30a^5}{30a^5}$	2) $\frac{10a^{12}}{5a^{12}}$
3) $(-3x)^0$	4) $-3x^0$

18. **Part II: Negative Exponents:** Directions: Simplify each expression. Show all work *or* explain what you did. ***Final answers should only contain positive exponents.***

5) $(3x)^{-5}$	6) $7x^{-3}$
7) $\frac{a^7}{a^6}$	8) $\frac{a^6}{a^7}$
9) $90d^5 \div 9d^8$	10) $\frac{4b^2c^5d}{10b^4c^3d^2}$

19. **Part III: Mixed Review** Directions: Simplify each expression. Show all work *or* explain what you did. You many need to use more than one rule per question (product rule, power to a power rule, quotient rule). ***Final answers should only contain positive exponents.***

11) $\frac{(6g^4)^2}{4g^{10}}$	12) $\frac{3k \bullet 8k^3}{12k^4}$
13) $\frac{3xy^2}{24x^4y^2}$	14) $\frac{-5a^3c^3}{20a^3c^4d^2}$
15) $\frac{9ab^5 \bullet 2a^4b^7}{18a^5b^{12}}$	16) $\left(\frac{15x^9}{5x^6}\right)^0$

20. Evaluating with negative exponents.

Lesson 4.2

1. Write as a single power. Express your answers with positive exponents.

a) $5(5^4)$

b) $\dfrac{(-8)^4}{(-8)^5}$

c) $(9^3)^6$

d) $\dfrac{3(3)^6}{3^5}$

e) $\left(\dfrac{1}{10}\right)^6\left(\dfrac{1}{10}\right)^{-4}$

f) $\left(\dfrac{(7)^2}{(7)^4}\right)^{-1}$

2. Evaluate. Express answers in rational form.

a) $4^{-2} - 8^{-1}$

b) $(4 + 8)^0 - 5^{-2}$

c) $25^{-1} + 3(5^{-1})^2$

d) $\left(-\dfrac{1}{2}\right)^3 + 4^{-3}$

3. Evaluate. Express answers in rational form.

a) $\left(\dfrac{4}{7}\right)^2$

b) $\left(-\dfrac{2}{5}\right)^3$

c) $\left(\dfrac{-2}{3}\right)^{-3}$

d) $\dfrac{(-3)^{-2}}{(-3)^{-5}}$

9. Simplify. Express answers with positive exponents.

a) $\dfrac{(x^{-3})x^5}{x^7}$

b) $\dfrac{(n^{-4})n^{-6}}{(n^{-2})^7}$

c) $\left(\dfrac{(y^2)^6}{y^9}\right)^{-2}$

d) $\dfrac{(-2x^5)^3}{8x^{10}}$

e) $(3a^2)^{-3}(9a^{-1})^2$

f) $\dfrac{(4r^{-6})(-2r^2)^5}{(-2r)^4}$

11. Evaluate each expression for $a = 2$ and $b = 3$. Express values in rational form.

a) $\left(\dfrac{b^3}{a^{\frac{5}{2}}}\right)^2\left(\dfrac{2a^4}{b^5}\right)$

b) $\sqrt{\dfrac{9b^3(ab)^2}{(a^2b^3)^3}}$

12. Simplify.

a) $(a^{10+2p})(a^{-p-8})$

b) $(2x^2)^{3-2m}\left(\dfrac{1}{x}\right)^{2m}$

c) $[(c)^{2n-3m}](c^3)^m \div (c^2)^n$

d) $(x^{4n-m})\left(\dfrac{1}{x^3}\right)^{m+n}$