IB Mathematics HL—Year 1
Unit 6: Worksheet—IB Style Questions
(Core Topic 7)

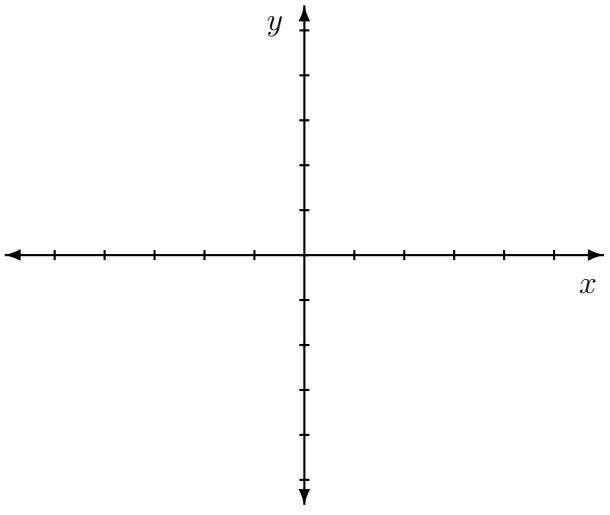
Name: _____ **Date:**_____ **Period:** ____

1. The function f is given with $f''(x) = 2x - 2$. When the graph of f is drawn, it has a minimum point at $(3, -7)$.

   (a) Show that $f'(x) = x^2 - 2x - 3$, and then find $f(x)$ explicitly.

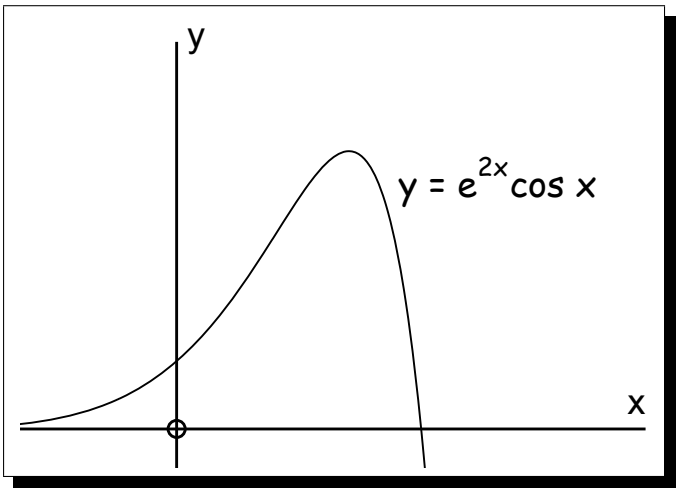
   (b) find $f(0)$, $f(-1)$, and $f'(-1)$.

   (c) Sketch the graph of f, labelling it with the information obtained in part (b).



Graph of $y = f(x)$

2. The diagram to the right shows part of the graph of the curve with equation $y = e^{2x} \cos x$.



(a) Show that $\dfrac{dy}{dx} = e^{2x}(2\cos x - \sin x)$.

(b) Find $\dfrac{d^2y}{dx^2}$

There is an inflection point at $P(a, b)$.

(c) Use the results from parts (a) and (b) to prove that

(i) $\tan a = \dfrac{3}{4}$.

(ii) the gradient of the curve at P is e^{2a}.

3. The function f is given by $f(x) = \dfrac{\ln 2x}{x}, \quad x > 0.$

(a) (i) Show that $f'(x) = \dfrac{1 - \ln 2x}{x^2}$.

Hence,

(ii) prove that the graph of f can have only one local maximum or minimum point;

(iii) find the coordinates of the maximum point on the graph of f.

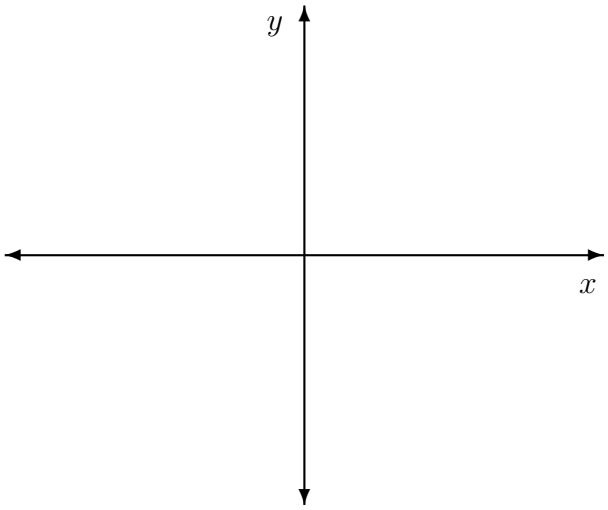
(b) Find the coordinates of the point of inflection on the graph of f.

4. Let f be the function defined by setting $f(x) = \dfrac{2}{1+x^3}$.

(a) (i) Write down the equation of the vertical asymptote of the graph of f.

(ii) Write down the equation of the horizontal asymptote of the graph of f.

(iii) Sketch the graph of f in the domain $-3 \leq x \leq 3$, indicating the asymptotes.



Graph of $y = f(x)$

(b) Find the coordinates of the point(s) of inflection of the graph of f and indicate them on the graph above.

5. The function g is defined by setting $g(x) = \dfrac{x^2}{2^x}, \quad x > 0.$

(a) Show that $g'(x) = \dfrac{2x - x^2 \ln 2}{2^x}$.

(b) Find the **exact** solution of $g'(x) = 0$.

(c) Show that the value of x obtained in (b) gives a maximum value for g.

6. The function g is defined for $-3 \leq x \leq 3$. The behavior of g' and g'' is given in the tables below.

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	positive	0	negative

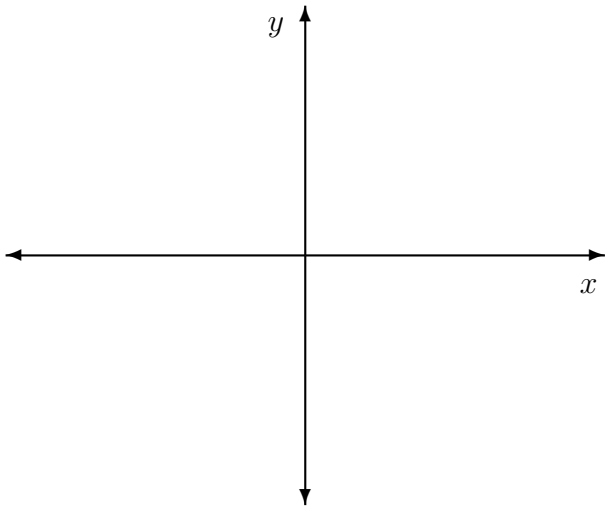
Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of x for which g has a maximum.

(b) On which intervals is g increasing?

(c) Write down the value of x for which the graph of g has a point of inflection.

(d) Given that $g(-3) = 1,\ g(-2) = -2,\ g(-1/2) = -1,$ and $g(1) = 0,$ sketch a graph of g consistent with the above information. On your sketch, indicate the maximum point, the minimum point, and the point of inflection.



7. A point $P(x, x^2)$ lies on the curve $y = x^2$. Calculate the minimum distance from the point $A\left(2, -\frac{1}{2}\right)$ to the point P.

8. A rectangle is drawn so that its lower vertices are at the points $(\pm x, 0)$ and its upper vertices are on the curve $y = e^{-x^2}$. The area of this rectangle is denote by A.

   (a) Write down an expression of A in terms of x.

   (b) Find the maximum value of A.

9. Find the maximum value of the function f on the interval $[-1, 3]$, where $f(x) = x^3 - 10x^2 + 12x + 23$.