IB Mathematics HL

Year I—Unit 6 Test

Name: ______________________________ **Date:**____ **Period:** ___

Part 1, No Calculators.

1. Let $f(x) = \tan^{-1} x^2$.

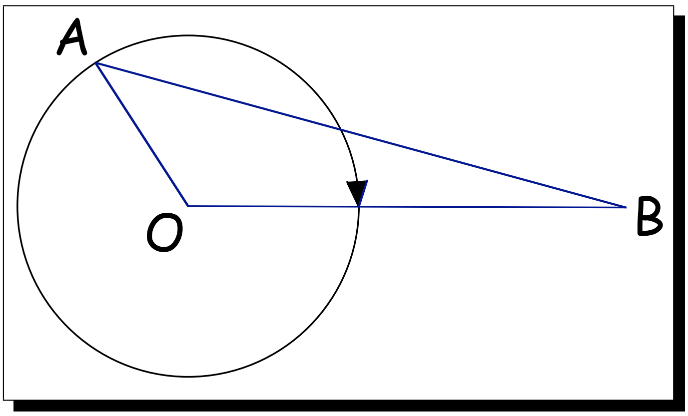
   (a) **(3 points)** Compute $f'(x)$.

   (b) **(4 points)** Find the equation of line tangent to the graph of $y = f(x)$ where $x = 1$.

2. **(7 points)** The function f is given by $f(x) = \dfrac{3}{x^3+1}$, $x > 0$. There is a point of inflection on the graph of f at the point P. Find the coordinates of this point.

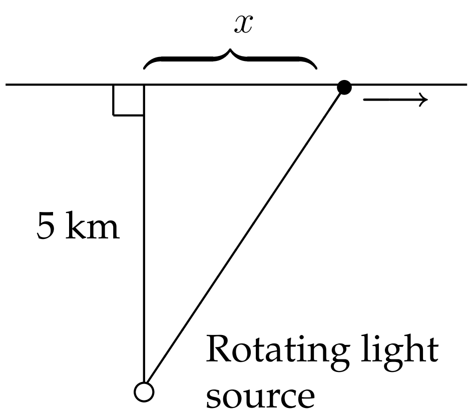
3. **(8 points)** Find the area of the largest rectangle that can be drawn under the graph of $y = 4 - x^2$ and above the x-axis.

4. **(8 points)** The figure below shows a shaft AB is 30 cm long and which is attached to a flywheel at the point A. The point B is constrained to move along the shaft OX. The radius of the wheel is 15 cm, and the wheel rotates at 100 revolutions per second. Find the rate of change in angle $\angle ABO$ when $\angle AOB$ is 120°.



5. **(6 points)** Assume that $y^2(5-x) = x^4$. Find the possible values of $\dfrac{dy}{dx}$ where $x = 1$.

6. **(8 points)** A powerful rotating light souce is positioned 5 km away from a coastline, as indicated in the picture. The light is rotating at a rate of 6 revolutions per minute in the clockwise direction. Therefore, during a certain portion of the light's revolution, the light will meet this coastline, and will move from left to right.



How fast (in km/min) is the point of intersection of the light with the coastline moving at the time when this intersection is nearest the light source?

7. **(6 points)** Find the maximum and minimum values of the function $f(x) = 2xe^{-x^2/50}$, $x \geq -4$.

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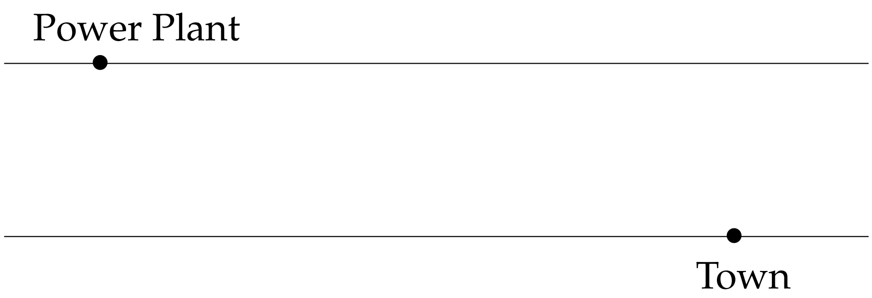
Part 2, Calculators allowed.

8. **(6 points)** A curve has equation $xy^3 + 2x^2y = 3$. Find the equation of the normal line to this curve at the point $(1, 1)$.

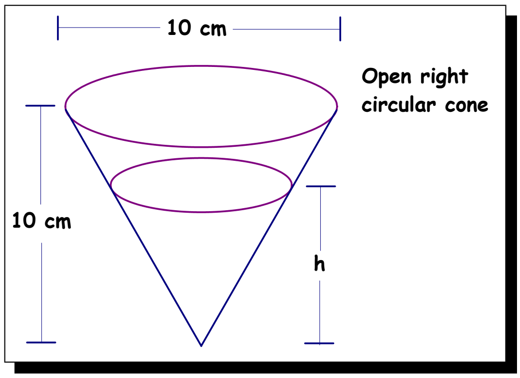
9. **(5 points)** How many points of inflection are there on the graph of $y = f(x)$, given that $-10 \leq x \leq 10$ and that $f''(x) = x \sin x - 1$. Explain your reasoning.

10. **(6 points)** Let $f(x) = x^3 \cos x,\ 0 \le x \le \dfrac{\pi}{2}$. Find the value of x for which f assumes its maximum value, explaining why this value of x gives a maximum.

11. **(8 points)** Suppose that on one side of a 2 km-wide river is an electricity-generating plant. On the opposite side, and 10 km down the river is a small town that will be consuming the electricity. If it costs \$80/m to lay cable under the river and \$40/m to lay cable over land, find the most strategic method for laying the cable.



12. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that is depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.



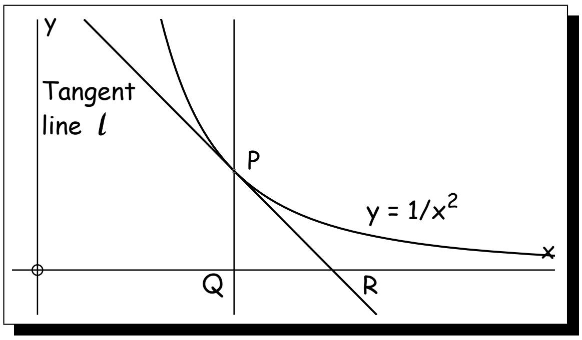
(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) **(2 points)** Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) **(3 points)** Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) **(4 points)** Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

13. In the figure to the right, line ℓ is tangent to the graph of $y = \dfrac{1}{x^2}$ at variable point P, with coordinates $\left(x, \dfrac{1}{x^2}\right)$, where $x > 0$. Point Q has coordinates $(x, 0)$. Line l crosses the x-axis at point R, with coordinates $(k, 0)$.



(a) **(3 points)** Find the value of k when $x = 3$.

(b) **(3 points)** For all $x > 0$, find k in terms of x.

(c) **(5 points)** Suppose that x is increasing at the constant rate of 7 units per second. When $x = 5$, what is the rate of change of k with respect to time?

(d) **(5 points)** Suppose that x is increasing at the constant rate of 7 units per second. When $x = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

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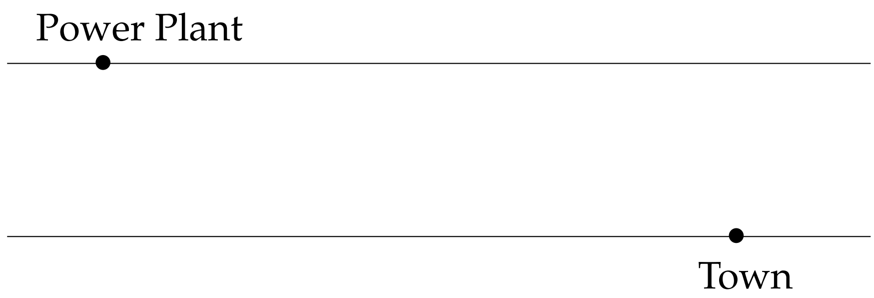

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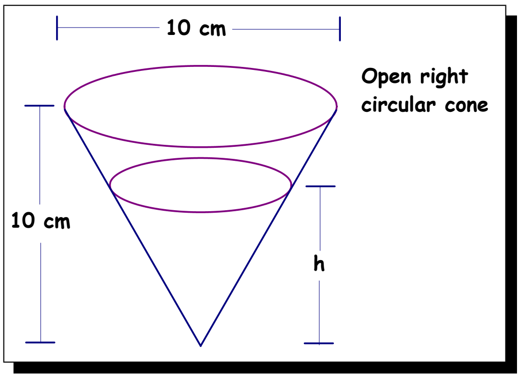

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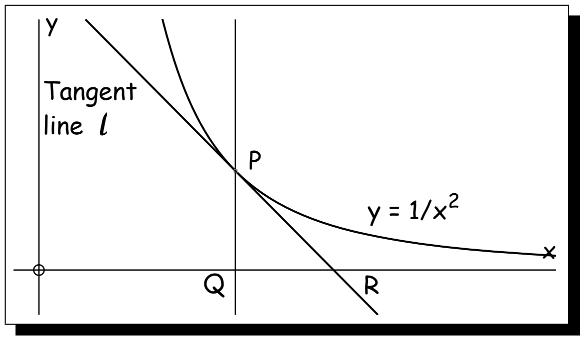

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