

IB Mathematics HL

Year I—Unit 5 Test

Name: _____ Date: _____ Period: _____

Instructions and Comments. No calculators allowed on any portion of this test. This test is worth 100 points.

1. **(8 points)** The line $y = 16x - 9$ is a tangent to the curve $y = 2x^3 + ax^2 + bx - 9$ at the point $(1, 7)$. Find the values of a and b .

2. **(8 points)** Given the function $s(x) = xe^{-x/10}$, determine the point $P(a, s(a))$, where $s'(a) = 0$.

3. Let $y = \frac{x}{1+x^2}$, where $x \geq 0$.

(a) **(4 points)** Compute $\frac{dy}{dx}$.

(b) **(5 points)** Determine the maximum value of y and determine for which value of x this maximum value occurs.

4. **(8 points)** Given that $y = \frac{1}{1+2e^{-x}}$, one can show that $\frac{dy}{dx} = y(1-y)$. (You don't need to show this.) Using this equation solve the equation $\frac{d^2y}{dx^2} = 0$ for x .

5. **(8 points)** The **normal** to the curve $y = \frac{k}{x} + \ln x^2$, $x \neq 0$, $k \in \mathbb{R}$, at the point where $x = 2$, has equation $3x + 2y = b$, where $b \in \mathbb{R}$. Find the **exact** value of k .

6. **(7 points)** Given that $y = x \ln(2x - 1)$, find $\frac{d^2y}{dx^2}$.

7. Consider the function defined by $y = \tan x - 8 \sin x$.

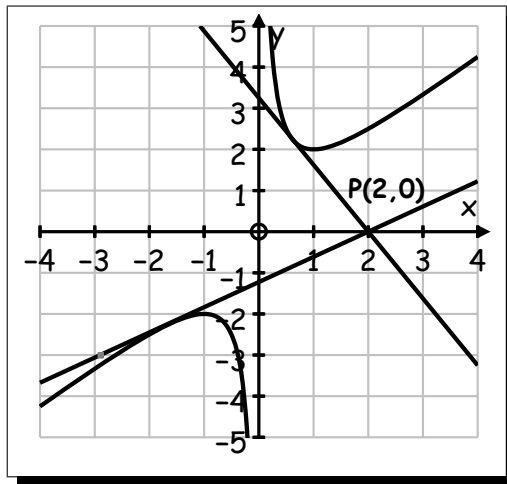
(a) **(4 points)** Find $\frac{dy}{dx}$.

(b) **(4 points)** Find the value of $\cos x$ for which $\frac{dy}{dx} = 0$.

8. (8 points) The figure to the right depicts the graph of

$$y = x + \frac{1}{x},$$

as well as two tangent lines to this graph, both passing through the point $P(2,0)$. Find the two slopes of these tangent lines.



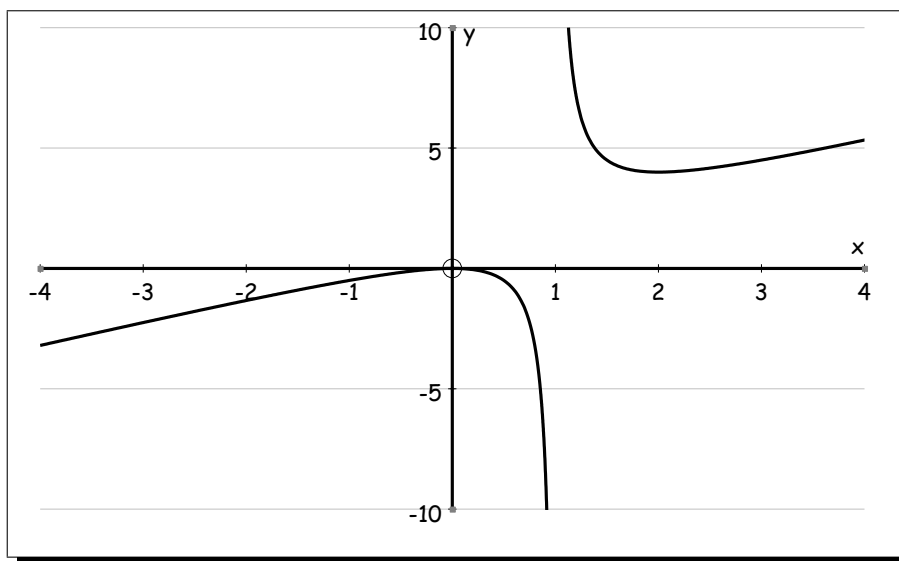
9. (8 points) Let $f(x) = x^3 + 3x^2 - 9x + 4$, and solve the inequality $f'(x) < 0$.

10. Let $g(x) = x^3 + 4x^2 + x - 6$.

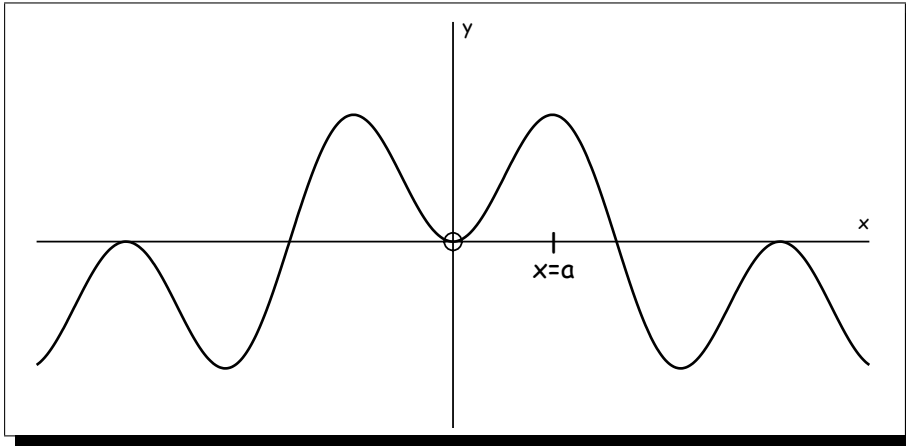
(a) **(4 points)** Find the equation of the tangent line ℓ to the graph of $y = g(x)$ where $x = -1$.

(b) **(5 points)** Find the coordinates of the point where ℓ meets the graph of $y = g(x)$ again.

11. **(8 points)** Given the graph of the rational function f below, give a sketch of the graph of f' on the same coordinate axes. (Note: the graph of $y = f(x)$ has a vertical asymptote at $x = 1$.)



12. Below is shown the graph of the function $f(x) = \sin^2 x \cos x$.



(a) (3 points) What is the period of the function f ?

(b) (4 points) You are given that at the indicated value $x = a$, the tangent to the graph is horizontal. Show that $\cos a = \sqrt{\frac{1}{3}}$.

(c) (4 points) Compute $f(a)$ exactly.