Permutations
and
Combinations

HL2 Math - Santowski

Objectives:

- apply fundamental counting principle
- compute permutations
- compute combinations
- distinguish permutations vs combinations

Fundamental Counting Principle

Fundamental Counting Principle can be used determine the number of possible outcomes when there are two or more characteristics .

Fundamental Counting Principle states that if an event has m possible outcomes and another independent event has n possible outcomes, then there are $m * n$ possible outcomes for the two events together.

Fundamental Counting Principle

Lets start with a simple example.

A student is to roll a die and flip a coin. How many possible outcomes will there be?

Fundamental Counting Principle

Lets start with a simple example.

A student is to roll a die and flip a coin. How many possible outcomes will there be?

1H	2H	3H	4H	5H	6H
1T	2T	3T	4T	5T	6T

6*2 = 12 outcomes

12 outcomes

Fundamental Counting Principle

For a college interview, Robert has to choose what to wear from the following: 4 slacks, 3 shirts, 2 shoes and 5 ties. How many possible outfits does he have to choose from?

Fundamental Counting Principle

For a college interview, Robert has to choose what to wear from the following: 4 slacks, 3 shirts, 2 shoes and 5 ties. How many possible outfits does he have to choose from?

4*3*2*5 = 120 outfits

Permutations

A **Permutation** is an arrangement of items in a particular order.

Notice, ORDER MATTERS!

To find the number of Permutations of n items, we can use the Fundamental Counting Principle or factorial notation.

Permutations

The number of ways to arrange the letters ABC:

	___	___	___
Number of choices for first blank?	3	___	___
Number of choices for second blank?	3	2	___
Number of choices for third blank?	3	2	1

3*2*1 = 6 3! = 3*2*1 = 6

ABC ACB BAC BCA CAB CBA

Permutations

To find the number of Permutations of n items chosen r at a time, you can use the formula

$_5P_3$

Permutations

To find the number of Permutations of n items chosen r at a time, you can use the formula

$$_nP_r = \frac{n!}{(n-r)!} \quad \text{where } 0 \le r \le n.$$

$$_5P_3 = \frac{\mathbf{5!}}{\mathbf{(5-3)!}} = \frac{5!}{2!} = \mathbf{5*4*3 = 60}$$

Permutations

Practice:

A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected. How many different lock combinations are possible assuming no number is repeated?

Permutations

Practice:

A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected. How many different lock combinations are possible assuming no number is repeated?

$$_{30}P_3 = \frac{\mathbf{30!}}{\mathbf{(30-3)!}} = \frac{30!}{27!} = \mathbf{30 * 29 * 28 = 24360}$$

Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

$$_{24}P_5 = \frac{\mathbf{24!}}{\mathbf{(24-5)!}} = \frac{24!}{19!} =$$

$$\mathbf{24 * 23 * 22 * 21 * 20 = 5{,}100{,}480}$$

Combinations

A **Combination** is an arrangement of items in which order does not matter.

ORDER DOES NOT MATTER!

Since the order does not matter in combinations, there are fewer combinations than permutations. The combinations are a "subset" of the permutations.

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$$_nC_r = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \le r \le n.$$

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$_5C_3$

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$$_{n}C_{r} = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \le r \le n.$$

$$_{5}C_{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} =$$

$$\frac{5*4*3*2*1}{3*2*1*2*1} = \frac{5*4}{2*1} = \frac{20}{2} = 10$$

Combinations

Practice:

To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

Combinations

Practice: To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

$$_{52}C_{5} = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} =$$

$$\frac{52*51*50*49*48}{5*4*3*2*1} = 2{,}598{,}960$$

Combinations

Practice:

A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

Combinations

Practice: A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

$$_{5}C_{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5*4}{2*1} = 10$$

Combinations

Practice:

A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Combinations

Practice: A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Center: $_2C_1 = \frac{2!}{1!1!} = 2$

Forwards: $_5C_2 = \frac{5!}{2!3!} = \frac{5*4}{2*1} = 10$

Guards: $_4C_2 = \frac{4!}{2!2!} = \frac{4*3}{2*1} = 6$

$$_2C_1 * {}_5C_2 * {}_4C_2$$

Thus, the number of ways to select the starting line up is 2*10*6 = 120.

Guidelines on Which Method to Use

Permutations	Combinations
Number of ways of selecting r items out of n items	
Repetitions are not allowed	
Order is important.	Order is not important.
Arrangements of n items taken r at a time	Subsets of n items taken r at a time
$_nP_r = n!/(n-r)!$	$_nC_r = n!/[\,r!(n-r)!]$
Clue words: arrangement, schedule, order	Clue words: group, sample, selection