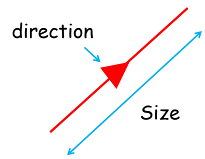
Lesson 83 – Algebraic Vectors

HL Math - Santowski

What do they look like?

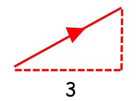
Vectors can be represented as:

-Straight Lines
(directed lines segments)

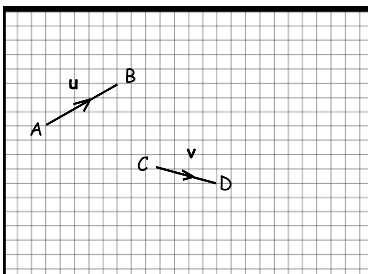


- Column Vectors

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



➔ Vectors are pathways



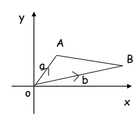
$\overrightarrow{AB}$ = **u** = $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\overrightarrow{CD}$ = **v** = $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

These are known as column vectors.

Position Vectors

$\overrightarrow{OA}$ is the journey from the origin to the point A. It is known as the position vector written **a**

$\overrightarrow{OB}$ Is the position vector of the point b, written **b**.



$\overrightarrow{AB}$ = **b** - **a** where **a** and **b** are the position vectors of A and B

<u>Example</u>
If P and Q have coordinates (4,8) and 2,3), respectively, find the components of $\overrightarrow{PQ}$.

$\overrightarrow{PQ}$ = **q** - **p**

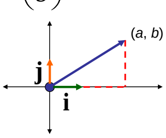
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

Vectors are denoted with **bold** letters

$$\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$$

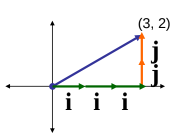
This is the notation for a position vector. This means the point (a, b) is the terminal point and the initial point is the origin.

$$\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix} = a\mathbf{i} + b\mathbf{j}$$



We use vectors that are only 1 unit long to build position vectors. **i** is a vector 1 unit long in the *x* direction and **j** is a vector 1 unit long in the *y* direction.

$$\mathbf{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$



$$\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$$

If we want to add vectors that are in the form *a***i** + *b***j**, we can just add the **i** components and then the **j** components.

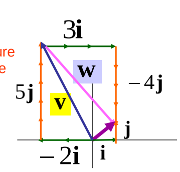
$$\mathbf{v} = -2\mathbf{i} + 5\mathbf{j} \qquad \mathbf{w} = 3\mathbf{i} - 4\mathbf{j}$$

$$\mathbf{v} + \mathbf{w} = -2\mathbf{i} + 5\mathbf{j} + 3\mathbf{i} - 4\mathbf{j} = \mathbf{i} + \mathbf{j}$$

When we want to know the magnitude of the vector (remember this is the length) we denote it

Let's look at this geometrically:

Can you see from this picture how to find the length of **v**?



$$\|\mathbf{v}\| = \sqrt{(-2)^2 + (5)^2}$$

$$= \sqrt{29}$$

(A) Operations with Algebraic Vectors

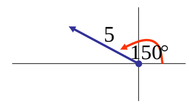
- Oxford 11D, p567, Q4
- Oxford 11G, p579, Q2,3,5

- Dunkley, Chap 1.6, p26, Q2f,4c,15

If we know the magnitude and direction of the vector, let's see if we can express the vector in *a***i** + *b***j** form.

$$\|\mathbf{v}\| = 5,\ \alpha = 150°$$

As usual we can use the trig we know to find the length in the horizontal direction and in the vertical direction.



$$\boxed{\mathbf{v} = \|\mathbf{v}\|\left(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}\right)}$$

$$\mathbf{v} = 5(\cos 150°\mathbf{i} + \sin 150°\mathbf{j}) = -\frac{5\sqrt{3}}{2}\mathbf{i} + \frac{5}{2}\mathbf{j}$$

(B) Magnitude and Direction of Vectors

- (1) Given the point P(3,7), determine the length and direction of the position vector $\overrightarrow{OP}$

- (2) How would you do the same Q for a vector in $\mathbf{R}^3$, using the point P(3,7,2)?

(B) Magnitude and Direction of Vectors

- (3) Find the angle between the vectors **u** = 2**i**+3**j** and the vector **v** = 5**i**+**j**

- (4) Given the three points A(2,1,3), B(3,5,1) and C(4,3,6), determine:
  - (i) the perimeter of this triangle
  - (ii) HENCE, use Herron's Formula to find its area
  - (iii) HENCE, determine the measure of all three of the interior angles.

(C) Collinearity

- (1) Use vectors to demonstrate that these points are collinear:
  - (i) P(15,10), Q(6,4), R(-12,-8)
  - (ii) D(33,-5,20), E(6,4,-16), F(9,3,-12)

- (2) Oxford, 11F, p573, Q3,4,6,7

A **unit vector** is a vector with magnitude 1.

If we want to find the unit vector having the same direction as a given vector, we find the magnitude of the vector and divide the vector by that value.

$$\mathbf{w} = 3\mathbf{i} - 4\mathbf{j} \qquad \text{What is } \|\mathbf{w}\|?$$

$$\|\mathbf{w}\| = \sqrt{(3)^2 + (-4)^2} = \sqrt{25} = 5$$

If we want to find the unit vector having the same direction as **w** we need to divide **w** by 5.

$$\mathbf{u} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$$

Let's check this to see if it really is 1 unit long.

$$\|\mathbf{u}\| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2} = \sqrt{\frac{25}{25}} = 1$$

(D) Unit Vectors

- (1) Oxford 11E, p570, Q1,2,4,7
- (2) If **v** = (3,4,12), find a unit vector in the same direction as **v**.
- (3) Oxford 11G, p579, Q4