Lesson 78 – Polar Form of Complex Numbers

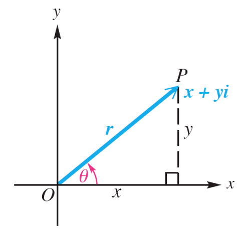
HL2 Math - Santowski

Relationships Among x, y, r, and θ

- $x = r\cos\theta$
  $y = r\sin\theta$

  $r = \sqrt{x^2 + y^2}$

  $\tan\theta = \dfrac{y}{x}, \quad \text{if } x \neq 0$



Polar Form of a Complex Number

- The expression $r(\cos\theta + i\sin\theta)$

  is called the **polar form** (or **trigonometric form**) of the complex number $x + yi$. The expression $\cos\theta + i\sin\theta$ is sometimes abbreviated cis θ.

Using this notation $r(\cos\theta + i\sin\theta)$ is written r cis θ.

Example

- Express $2(\cos 120° + i\sin 120°)$ in rectangular form.

Example

- Express 2(cos 120° + *i* sin 120°) in rectangular form.

$$\cos 120^\circ = -\frac{1}{2} \qquad 2(\cos 120^\circ + i\sin 120^\circ) = 2\left(-\frac{1}{2}, i\frac{\sqrt{3}}{2}\right)$$

$$= -1 + i\sqrt{3}$$

$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

- Notice that the real part is negative and the imaginary part is positive, this is consistent with 120 degrees being a quadrant II angle.

Converting from Rectangular to Polar Form

- Step 1 Sketch a graph of the number $x + yi$ in the complex plane.
- Step 2 Find r by using the equation $r = \sqrt{x^2 + y^2}$.
- Step 3 Find θ by using the equation $\tan\theta = \frac{y}{x}, x \neq 0$ choosing the quadrant indicated in Step 1.

Example

- Example: Find trigonometric notation for -1 - *i*.

Example

- Example: Find trigonometric notation for -1 - *i*.
- First, find r.

$$r = \sqrt{a^2 + b^2} \qquad \sin\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \qquad \cos\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$r = \sqrt{(-1)^2 + (-1)^2} \qquad \theta = \frac{5\pi}{4}$$

$$r = \sqrt{2}$$

- Thus, $-1 - i = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ or $\sqrt{2}\ \text{cis}\ \frac{5\pi}{4}$

Product of Complex Numbers

- Find the product of
  $4(\cos 50^\circ + i\sin 50^\circ)$ and $2(\cos 10^\circ + i\sin 10^\circ)$.

Product Theorem

- If $r_1 = (\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$, are any two complex numbers, then

$$\left[r_1(\cos\theta_1 + i\sin\theta_1)\right]\cdot\left[r_2(\cos\theta_2 + i\sin\theta_2)\right]$$
$$= r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right].$$

- In compact form, this is written

$$(r_1 \text{ cis } \theta_1)(r_2 \text{ cis } \theta_2) = r_1 r_2 \text{ cis}(\theta_1+\theta_2).$$

Example: Product

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$$\left[4(\cos 50^\circ + i\sin 50^\circ)\right]\cdot\left[2(\cos 10^\circ + i\sin 10^\circ)\right]$$
$$= 4\cdot 2\left[\cos(50^\circ + 10^\circ) + i\sin(50^\circ + 10^\circ)\right]$$
$$= 8(\cos 60^\circ + i\sin 60^\circ)$$
$$= 8\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$
$$= 4 + 4i\sqrt{3}$$

Quotient of Complex Numbers

- Find the quotient.

$16(\cos 70^\circ + i\sin 70^\circ)$ and $4(\cos 40^\circ + i\sin 40^\circ)$

Quotient Theorem

- If $r_1 = (\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, where

$r_2(\cos\theta_2 + i\sin\theta_2),\ r_2 \neq 0,$

then

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right].$$

In compact form, this is written

$$\frac{r_1 \text{ cis } \theta_1}{r_2 \text{ cis } \theta_2} = \frac{r_1}{r_2} \text{ cis}(\theta_1 - \theta_2)$$

Example: Quotient

- Find the quotient.

$16(\cos 70^\circ + i\sin 70^\circ)$ and $4(\cos 40^\circ + i\sin 40^\circ)$

Example: Quotient

- Find the quotient.

$16(\cos 70^\circ + i\sin 70^\circ)$ and $4(\cos 40^\circ + i\sin 40^\circ)$

$$\begin{aligned}\frac{16(\cos 70^\circ + i\sin 70^\circ)}{4(\cos 40^\circ + i\sin 40^\circ)} &= \frac{16}{4}\left(\cos(70^\circ - 40^\circ) + i\sin(70^\circ - 40^\circ)\right)\\ &= 4\cos 30^\circ + i\sin 30^\circ\\ &= 4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\\ &= 2\sqrt{3} + 2i\end{aligned}$$

If $z = 4\left(\cos 40^\circ + i\sin 40^\circ\right)$ and $w = 6\left(\cos 120^\circ + i\sin 120^\circ\right)$,
find: (a) zw (b) z/w

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$$zw = \left[4\left(\cos 40^\circ + i\sin 40^\circ\right)\right]\left[6\left(\cos 120^\circ + i\sin 120^\circ\right)\right]$$

$$= \left[(4\cdot 6)\left(\cos(40^\circ + 120^\circ) + i\sin(40^\circ + 120^\circ)\right)\right]$$

multiply the moduli

add the arguments (the *i* sine term will have same argument)

$$= 24\left(\cos 160^\circ + i\sin 160^\circ\right)$$
$$= 24(-0.93969 + 0.34202i)$$
$$= -22.55 + 8.21i$$

If you want the answer in rectangular coordinates simply compute the trig functions and multiply the 24 through.

$$\frac{z}{w} = \frac{4\left(\cos 40^\circ + i\sin 40^\circ\right)}{6\left(\cos 120^\circ + i\sin 120^\circ\right)}$$

$$= \frac{4}{6}\left[\cos\left(40^\circ - 120^\circ\right) + i\sin\left(40^\circ - 120^\circ\right)\right]$$

divide the moduli

subtract the arguments

$$= \frac{2}{3}\left[\cos\left(-80^\circ\right) + i\sin\left(-80^\circ\right)\right]$$

$$= \frac{2}{3}\left[\cos\left(280^\circ\right) + i\sin\left(280^\circ\right)\right]$$

In polar form we want an angle between 0 and 360° so add 360° to the -80°

In rectangular coordinates: $= \frac{2}{3}(0.1736 - 0.9848i) = 0.12 - 0.66i$

Example

divide $\frac{z_1}{z_2}$

Where $z_1 = 3\sqrt{2} + 3\sqrt{2}i = 6(\cos 45 + i\sin 45)$

$z_2 = 2\sqrt{3} + 2i = 4(\cos 30 + i\sin 30)$

Rectangular form

Trig form

Example

$$divide\ \frac{z_1}{z_2}$$

$$Where\ \ z_1 = 3\sqrt{2} + 3\sqrt{2}i = 6\left(\cos 45 + i\sin 45\right)$$

$$z_2 = 2\sqrt{3} + 2i = 4\left(\cos 30 + i\sin 30\right)$$

Rectangular form

$$\frac{3\sqrt{2} + 3\sqrt{2}i}{2\sqrt{3} + 2i}$$

$$\left(\frac{3\sqrt{2} + 3\sqrt{2}i}{2\sqrt{3} + 2i}\right)\left(\frac{2\sqrt{3} - 2i}{2\sqrt{3} - 2i}\right)$$

$$\frac{6\sqrt{6} - 6\sqrt{2}i + 6\sqrt{6}i - 6\sqrt{2}i^2}{12 - 4i^2}$$

$$\frac{6\sqrt{6} - 6\sqrt{2}i + 6\sqrt{6}i + 6\sqrt{2}}{12 + 4}$$

$$\frac{\left(6\sqrt{6} + 6\sqrt{2}\right)}{16} + \frac{\left(6\sqrt{6} - 6\sqrt{2}\right)i}{16}$$

Trig form

$$\frac{6\left(\cos 45 + i\sin 45\right)}{4\left(\cos 30 + i\sin 30\right)}$$

$$\frac{6}{4}\left(\cos\left(45 - 30\right) + i\sin\left(45 - 30\right)\right)$$

$$\frac{3}{2}\left(\cos 15 + i\sin 15\right)$$

$$r = \sqrt{\left(\frac{\left(6\sqrt{6} + 6\sqrt{2}\right)}{16}\right)^2 + \left(\frac{\left(6\sqrt{6} - 6\sqrt{2}\right)}{16}\right)^2} \qquad \theta = \tan^{-1}\left(\frac{\frac{\left(6\sqrt{6} - 6\sqrt{2}\right)}{16}}{\frac{\left(6\sqrt{6} + 6\sqrt{2}\right)}{16}}\right) = 15$$

$$r = \sqrt{\frac{216 + 72\sqrt{12} + 72 + 216 - 72\sqrt{12} + 72}{256}}$$

$$r = \sqrt{\frac{576}{256}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

DeMoivre's Theorem

Taking Complex Numbers to Higher Powers

$$z = r\left(\cos\theta + i\sin\theta\right)$$

$$z^2 = ??$$

$$z = r\left(\cos\theta + i\sin\theta\right)$$

$$z^2 = r\left(\cos\theta + i\sin\theta\right)r\left(\cos\theta + i\sin\theta\right)$$

$$= r^2\left(\cos^2\theta + 2i\cos\theta\sin\theta + i^2\sin\theta\right)$$

$$= r^2\left(\cos^2\theta - \sin^2\theta + i2\sin\theta\cos\theta\right)$$

$$= r^2\left(\cos 2\theta + i\sin 2\theta\right)$$

$$\boxed{z^2 = r^2\left(\cos 2\theta + i\sin 2\theta\right)}$$

Nice ☺

What about z^3?

$z = r(\cos\theta + i\sin\theta),\ z^2 = r^2(\cos 2\theta + i\sin 2\theta)$

$z^3 = z \cdot z^2 =$

$r^3(\cos 2\theta + i\sin 2\theta)(\cos\theta + i\sin\theta) =$

$r^3(\cos 2\theta\cos\theta + i\cos 2\theta\sin\theta + i\sin 2\theta\cos\theta + i^2\sin 2\theta\sin\theta) =$

$r^3(\cos 2\theta\cos\theta - \sin 2\theta\sin\theta + i\cos 2\theta\sin\theta + i\sin 2\theta\cos\theta) =$

$r^3[\cos 2\theta\cos\theta - \sin 2\theta\sin\theta + i(\sin 2\theta\cos\theta + \cos 2\theta\sin\theta)] =$

$r^3[\cos(2\theta+\theta) + i(\sin(2\theta+\theta))] = r^3(\cos 3\theta + i\sin 3\theta)$

$\boxed{z^3 = r^3(\cos 3\theta + i\sin 3\theta)}$

☺ Hooray!! ☺

We saw…

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$$z^2 = r^2(\cos 2\theta + i\sin 2\theta) \text{ and}$$

$$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

Similarly…

$$z^4 = r^4(\cos 4\theta + i\sin 4\theta)$$

$$z^5 = r^5(\cos 5\theta + i\sin 5\theta)$$

$$\vdots$$

$$\boxed{z^n = r^n(\cos n\theta + i\sin n\theta)}$$

Hooray DeMoivre and his incredible theorem

Powers of Complex Numbers

This is horrible in rectangular form.

$(a+bi)^n$

$(a+bi)(a+bi)(a+bi)...(a+bi)$

The best way to expand one of these is using Pascal's triangle and binomial expansion.

You'd need to use an i-chart to simplify.

It's much nicer in trig form. You just raise the r to the power and multiply theta by the exponent.

$z = r(\cos\theta + i\sin\theta)$

$z^n = r^n(\cos n\theta + i\sin n\theta)$

Example

$z = 5(\cos 20 + i\sin 20)$

$z^3 = 5^3(\cos 3\cdot 20 + i\sin 3\cdot 20)$

$z^3 = 125(\cos 60 + i\sin 60)$

De Moivre's Theorem

- If $r_1 = (\cos\theta_1 + i\sin\theta_1)$ is a complex number, and if n is any real number, then

$$\left[r(\cos\theta_1 + i\sin\theta_1)\right]^n = r^n(\cos n\theta + i\sin n\theta).$$

- In compact form, this is written $\left[r \text{ cis}\theta\right]^n = r^n(\text{cis } n\theta)$.

Example: Find $(-1 - i)^5$ and express the result in rectangular form.

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- First, find trigonometric notation for $-1 - i$

$$-1 - i = \sqrt{2}\left(\cos 225 + i \sin 225\right)$$

- Theorem

$$\begin{aligned}(-1-i)^5 &= \left[\sqrt{2}\left(\cos 225^\circ + i \sin 225^\circ\right)\right]^5 \\ &= \left(\sqrt{2}\right)^5\left[\cos(5 \cdot 225^\circ) + i \sin(5 \cdot 225^\circ)\right] \\ &= 4\sqrt{2}\left(\cos 1125^\circ + i \sin 1125^\circ\right) \\ &= 4\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) \\ &= 4 + 4i\end{aligned}$$

Example

- Let $z = 1 - i$ Find z^{10}

Example

Example 8.3.2* Let $z = 1 - i$. Find z^{10}.

Solution: First write z in polar form.

$|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\arg(z) = -\frac{\pi}{4}$ (or $\frac{7\pi}{4}$)

Polar Form : $z = \sqrt{2}\left(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})\right)$.

Applying de Moivre's Theorem gives :

$$\begin{aligned}z^{10} &= (\sqrt{2})^{10}\left(\cos(10 \times (-\frac{\pi}{4})) + i \sin(10 \times (-\frac{\pi}{4}))\right) \\ &= 2^5\left(\cos(-\frac{10\pi}{4}) + i \sin(-\frac{10\pi}{4})\right) \\ &= 32\left(\cos(-\frac{5\pi}{2}) + i \sin(-\frac{5\pi}{2})\right) \\ &= 32\left(\cos(-\frac{5\pi}{2} + 2\pi) + i \sin(-\frac{5\pi}{2} + 2\pi)\right) \\ &= 32\left(\cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2})\right) \\ &= 32(0 + i(-1)) \\ &= -32i\end{aligned}$$

Further Examples

In Exercises 35–44, use DeMoivre's Theorem to find the indicated powers of the given complex number. Express the result in standard form.

35. $(1 + i)^4$

36. $(2 + 2i)^6$

37. $(-1 + i)^{10}$

38. $(\sqrt{3} + i)^7$

39. $(1 - \sqrt{3}i)^3$

40. $\left[5\left(\cos\frac{\pi}{9} + i \sin\frac{\pi}{9}\right)\right]^3$

41. $\left[3\left(\cos\frac{5\pi}{6} + i \sin\frac{5\pi}{6}\right)\right]^4$

42. $\left(\cos\frac{5\pi}{4} + i \sin\frac{5\pi}{4}\right)^{10}$

43. $\left[2\left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}\right)\right]^8$

44. $\left[5\left(\cos\frac{3\pi}{2} + i \sin\frac{3\pi}{2}\right)\right]^4$

nth Roots

- For a positive integer n, the complex number $a + bi$ is an nth root of the complex number $x + yi$ if

$$(a+bi)^n = x + yi.$$

nth Root Theorem

- If n is any positive integer, r is a positive real number, and θ is in degrees, then the nonzero complex number $r(\cos\theta + i\sin\theta)$ has exactly n distinct nth roots, given by

- where $\sqrt[n]{r}\left(\cos\alpha + i\sin\alpha\right)$ or $\sqrt[n]{r}$ cis α,

$$\alpha = \frac{\theta + 360^\circ \cdot k}{n} \text{ or } \alpha = \frac{\theta}{n} + \frac{360^\circ \cdot k}{n}, \quad k = 0,1,2,\ldots,n-1.$$

Example: Square Roots

- Find the square roots of $1+\left(\sqrt{3}\right)$

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- Find the square roots of $1+\left(\sqrt{3}\right)$

- Polar notation: $1+\left(\sqrt{3}\right) = 2\left(\cos 60 + i\sin 60\right)$

$$\left[2\left(\cos 60 + i\sin 60\right)\right]^{\frac{1}{2}} = 2^{\frac{1}{2}}\left[\cos\left(\frac{60}{2} + k\cdot\frac{360}{2}\right) + i\sin\left(\frac{60}{2} + k\cdot\frac{360}{2}\right)\right]$$

$$= \sqrt{2}\left[\cos\left(30 + k\cdot 180\right) + i\sin\left(30 + k\cdot 180\right)\right]$$

- For $k = 0$, root is $\sqrt{2}\left(\cos 30 + i\sin 30\right)$
- For $k = 1$, root is $\sqrt{2}\left(\cos 210 + i\sin 210\right)$

Example: Fourth Root

- Find all fourth roots of $-8+8i\sqrt{3}$.

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Example: Fourth Root

- Find all fourth roots of $-8+8i\sqrt{3}$. Write the roots in rectangular form.
- Write in polar form. $-8+8i\sqrt{3} = 16 \text{ cis } 120^\circ$
- Here $r = 16$ and $\theta = 120°$. The fourth roots of this number have absolute value $\sqrt[4]{16} = 2$.

$$\alpha = \frac{120^\circ}{4} + \frac{360^\circ \cdot k}{4} = 30^\circ + 90^\circ \cdot k$$

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Example: Fourth Root continued

- There are four fourth roots, let $k = 0, 1, 2$ and 3.

$$\begin{array}{ll} k=0 & \alpha = 30^\circ + 90^\circ \cdot 0 = 30^\circ \\ k=1 & \alpha = 30^\circ + 90^\circ \cdot 1 = 120^\circ \\ k=2 & \alpha = 30^\circ + 90^\circ \cdot 2 = 210^\circ \\ k=3 & \alpha = 30^\circ + 90^\circ \cdot 3 = 300^\circ \end{array}$$

- Using these angles, the fourth roots are

$2 \text{ cis } 30^\circ$, $2 \text{ cis } 120^\circ$, $2 \text{ cis } 210^\circ$, $2 \text{ cis } 300^\circ$

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Example: Fourth Root continued

- Written in rectangular form

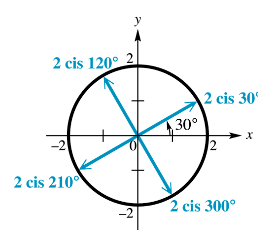
$\sqrt{3}+i$

$-1+i\sqrt{3}$

$-\sqrt{3}-i$

$1-i\sqrt{3}$

- The graphs of the roots are all on a circle that has center at the origin and radius 2.



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Find the complex fifth roots of $-2\sqrt{3}+2i$

Find the complex fifth roots of $-2\sqrt{3}+2i$

$$-2\sqrt{3}+2i = 4\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = 4(\cos 150^\circ + i\sin 150^\circ)$$

The five complex roots are:

$$z_k = \sqrt[5]{4}\left[\cos\left(\frac{150^\circ}{5}+\frac{360^\circ k}{5}\right)+i\sin\left(\frac{150^\circ}{5}+\frac{360^\circ k}{5}\right)\right]$$

$$\boxed{z_k = \sqrt[5]{4}\left[\cos(30^\circ + 72^\circ k)+i\sin(30^\circ + 72^\circ k)\right]}$$

for $k = 0, 1, 2, 3, 4$.

$$\boxed{z_k = \sqrt[5]{4}\left[\cos(30^\circ + 72^\circ k)+i\sin(30^\circ + 72^\circ k)\right]}$$

$$k=0,\quad z_0 = \sqrt[5]{4}\left[\cos(30^\circ)+i\sin(30^\circ)\right]$$

$$k=1,\quad z_1 = \sqrt[5]{4}\left[\cos(102^\circ)+i\sin(102^\circ)\right]$$

$$k=2,\quad z_2 = \sqrt[5]{4}\left[\cos(174^\circ)+i\sin(174^\circ)\right]$$

$$k=3,\quad z_3 = \sqrt[5]{4}\left[\cos(246^\circ)+i\sin(246^\circ)\right]$$

$$k=4,\quad z_4 = \sqrt[5]{4}\left[\cos(318^\circ)+i\sin(318^\circ)\right]$$

Example

- Find all the complex fifth roots of 32

$32 + 0i$

$r = \sqrt{a^2 + b^2} = \sqrt{32^2 + 0^2} = \sqrt{32^2} = 32$

$\theta = tan^{-1}\frac{0}{32} \qquad \theta = tan^{-1} 0 = 0°$

$32\left[cos\left(0°\right) + i\,sin\left(0°\right)\right]$

$\sqrt[5]{32}\left[cos\left(\frac{0}{5} + \frac{360k}{5}\right) + i\,sin\left(\frac{0}{5} + \frac{360k}{5}\right)\right]$

$2\left[cos\left(72k\right) + i\,sin\left(72k\right)\right]$

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$2\left[cos\left(72k\right) + i\,sin\left(72k\right)\right]$

$k = 0 \quad 2\left[cos\left(0°\right) + i\,sin\left(0°\right)\right] = 2\left(1 + 0i\right) = 2$

$k = 1 \quad 2\left[cos\left(72\right) + i\,sin\left(72\right)\right] = .62 + 1.90i$

$k = 2 \quad 2\left[cos\left(144\right) + i\,sin\left(144\right)\right] = -1.62 + 1.18i$

$k = 3 \quad 2\left[cos\left(216\right) + i\,sin\left(216\right)\right] = -1.62 - 1.18i$

$k = 4 \quad 2\left[cos\left(288\right) + i\,sin\left(288\right)\right] = .62 - 1.90i$

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Example Find $\sqrt[3]{i}$

You may assume it is the principle root you are seeking unless specifically stated otherwise.

First express i as a complex number in standard form. $0 + i$

Then change to polar form $r = \sqrt{a^2 + b^2}$ $\quad r = 1$

$tan\theta = \frac{b}{a} \qquad tan\theta = \frac{1}{0} \qquad \theta = tan^{-1}\frac{1}{0} \qquad \theta = 90°$

$1\left[cos\left(90°\right) + i\,sin\left(90°\right)\right]$

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$1\left[cos\left(90°\right) + i\,sin\left(90°\right)\right]$

Since we are looking for the cube root, use DeMoivre's Theorem and raise it to the $\frac{1}{3}$ power

$1^{\frac{1}{3}}\left[cos\left(\frac{1}{3}90°\right) + i\,sin\left(\frac{1}{3}90°\right)\right]$

$1\left[cos\left(30°\right) + i\,sin\left(30°\right)\right]$

$1\left[\frac{\sqrt{3}}{2} + i\frac{1}{2}\right] = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

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Example:

Find the 4th root of $-2+i$

$$\left(-2+i\right)^{\frac{1}{4}}$$

Change to polar form $\sqrt{5}\left[\cos\left(153.4\right)+i\sin\left(153.4\right)\right]$

Apply DeMoivre's Theorem

$$\sqrt{5}^{\frac{1}{4}}\left[\cos\left(\frac{1}{4}\right)\left(153.4\right)+i\sin\left(\frac{1}{4}\right)\left(153.4\right)\right]$$

$$1.22\left[\cos\left(38.4\right)+i\sin\left(38.4\right)\right]$$

$$.96+.76i$$

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