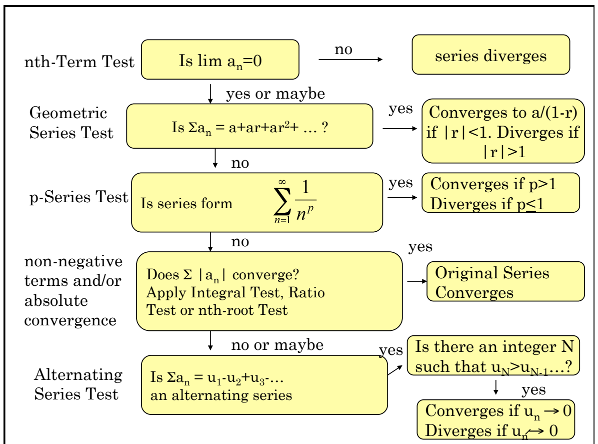
LESSON 70 – Alternating Series and Absolute Convergence & Conditional Convergence

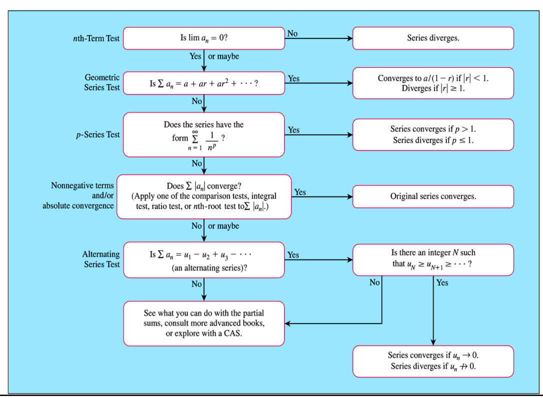
HL Math –Santowski

OBJECTIVES

- (a) Introduce and work with the convergence/ divergence of alternating series
- (b) Introduce and work with absolute convergence of series
- (c) Deciding on which method to use ……



PROCEDURE FOR DETERMINING CONVERGENCE



Alternating Series

A series in which terms alternate in sign

$$\sum_{n=1}^{\infty}(-1)^n a_n \quad \text{or} \quad \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2^n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \ldots$$

$$\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \ldots$$

TESTING CONVERGENCE - ALTERNATING SERIES TEST

Theorem The Alternating Series Test

The series

$$\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - \ldots$$

converges if all three of the following conditions are satisfied:

1. each u_n is positive;
2. $u_n \geq u_{n+1}$ for all $n \geq N$ for some integer N (decreasing);
3. $\lim_{n\to\infty} u_n \to 0$

TESTING CONVERGENCE - ALTERNATING SERIES TEST

Alternating Series

example: $$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$

TESTING CONVERGENCE - ALTERNATING SERIES TEST

Alternating Series

Alternating Series Test

If the absolute values of the terms approach zero, then an alternating series will always converge!

example: $$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$

This series converges (by the Alternating Series Test.)

This series is convergent, but not absolutely convergent.

Therefore we say that it is conditionally convergent.

EXAMPLES

- Investigate the convergence of the following series:

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n^2}$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n^3}$

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n+1}{n}$

- Show that the series $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$ converges

ABSOLUTE AND CONDITIONAL CONVERGENCE

- A series $\sum_{n=N}^{\infty} a_n$ is **absolutely convergent** if the corresponding series of absolute values $\sum_{n=N}^{\infty} |a_n|$ converges.
- A series that converges but does not converge absolutely, converges conditionally.
- Every absolutely convergent series converges. (Converse is false!!!)

IS THE GIVEN SERIES CONVERGENT OR DIVERGENT? IF IT IS CONVERGENT, ITS IT ABSOLUTELY CONVERGENT OR CONDITIONALLY CONVERGENT?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n(n+1)/2}}{3^n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

A) IS THE GIVEN SERIES CONVERGENT OR DIVERGENT? IF IT IS CONVERGENT, ITS IT ABSOLUTELY CONVERGENT OR CONDITIONALLY CONVERGENT?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n(n+1)/2}}{3^n} = -\frac{1}{3} - \frac{1}{9} + \frac{1}{27} + \frac{1}{81} - \ldots$$

This is not an alternating series, but since

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n(n+1)/2}}{3^n} \right| = \sum_{n=1}^{\infty} \frac{1}{3^n}$$

Is a convergent geometric series, then the given Series is <u>absolutely convergent</u>.

B) IS THE GIVEN SERIES CONVERGENT OR DIVERGENT? IF IT IS CONVERGENT, ITS IT ABSOLUTELY CONVERGENT OR CONDITIONALLY CONVERGENT?

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln(n+1)} = -\frac{1}{\ln 2}+\frac{1}{\ln 3}-\frac{1}{\ln 4}+\ldots\ldots$$

Converges by the Alternating series test.

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\ln(n+1)}\right| = \frac{1}{\ln 2}+\frac{1}{\ln 3}+\frac{1}{\ln 4}+\ldots\ldots$$

Diverges with direct comparison with the harmonic Series. The given series is conditionally convergent.

C) IS THE GIVEN SERIES CONVERGENT OR DIVERGENT? IF IT IS CONVERGENT, ITS IT ABSOLUTELY CONVERGENT OR CONDITIONALLY CONVERGENT?

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+1)}{n} = \frac{2}{1}-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+$$

By the nth term test for divergence, the series Diverges.

D) IS THE GIVEN SERIES CONVERGENT OR DIVERGENT? IF IT IS CONVERGENT, ITS IT ABSOLUTELY CONVERGENT OR CONDITIONALLY CONVERGENT?

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}} = -\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}$$

Converges by the alternating series test.

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt{n}}\right| = \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}$$

Diverges since it is a p-series with p <1. The Given series is conditionally convergent.

FURTHER EXAMPLES