Lesson 69 – Ratio Test & Comparison Tests

Math HL – Calculus Option

Series known to converge or diverge

]. A geometric series with | r | <1 converges

- 2. A repeating decimal converges
- 3. Telescoping series converge

A necessary condition for convergence: Limit as n goes to infinity for nth term in sequence is 0.

nth term test for divergence:

If the limit as n goes to infinity for the nth term is not 0, the series DIVERGES!

Convergent and Divergent Series

- If the infinite series has a sum, or limit, the series is convergent.
- \circ If the series is not convergent, it is divergent.

Ways To Determine Convergence/ Divergence

- 1. Arithmetic since no sum exists, it diverges
- o 2. Geometric:
 - If |r| > 1, diverges
 - If |r| < 1, converges since the sum exists
- \circ 3. Ratio Test (discussed in a few minutes)



Determine whether each series is convergent or divergent.

 $0 1/8 + 3/20 + 9/50 + 27/125 + \dots$

 \circ 18.75+17.50+16.25+15.00+ . . .

 $\circ 65 + 13 + 13/5 + 13/25 \dots$

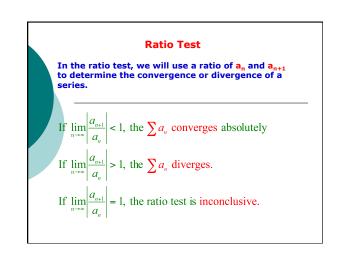


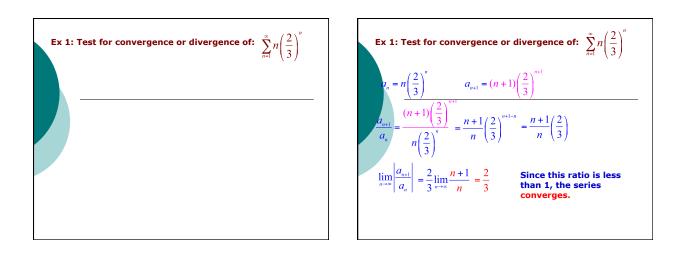
Determine whether each series is convergent or divergent.

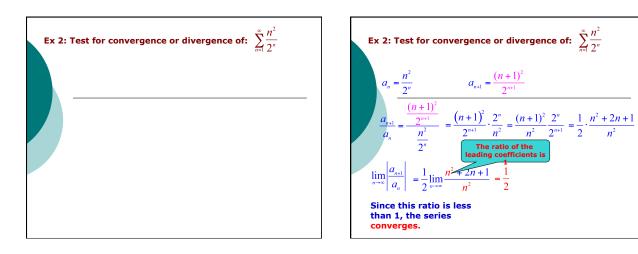
1/8 + 3/20 + 9/50 + 27/125 + ...
r=6/5 → |r|>1 → divergent
18.75+17.50+16.25+15.00+ ...
Arithmetic series → divergent
65 + 13 + 13/5 + 13/25 ...
r=1/5 → |r|<1 → convergent

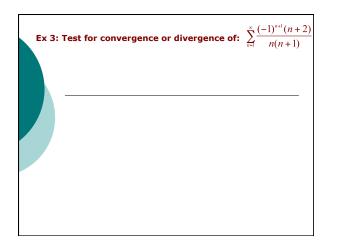


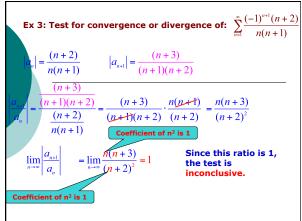
- When a series is neither arithmetic or geometric, it is more difficult to determine whether the series is convergent or divergent.
- So we need a variety of different analytical tools to help us decide whether a series converges or diverges





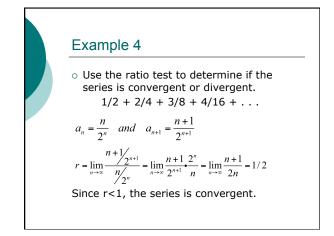






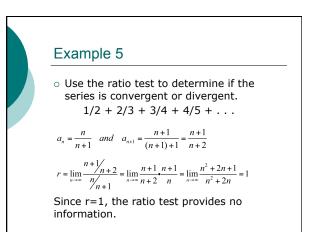
Example 4

• Use the ratio test to determine if the series is convergent or divergent. 1/2 + 2/4 + 3/8 + 4/16 + ...



Example 5

• Use the ratio test to determine if the series is convergent or divergent. 1/2 + 2/3 + 3/4 + 4/5 + ...



Example 6

• Use the ratio test to determine if the series is convergent or divergent. 2 + 3/2 + 4/3 + 5/4 + ...



• Use the ratio test to determine if the series is convergent or divergent. 2 + 3/2 + 4/3 + 5/4 + ...

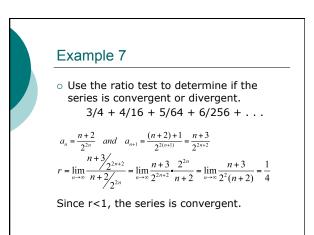
 $a_n = \frac{n+1}{n}$ and $a_{n+1} = \frac{(n+1)+1}{n+1} = \frac{n+2}{n+1}$

 $r = \lim_{n \to \infty} \frac{n+2/n+1}{n+1/n} = \lim_{n \to \infty} \frac{n+2}{n+1} \cdot \frac{n}{n+1} = \lim_{n \to \infty} \frac{n^2+2n}{n^2+2n+1} = 1$

Since r=1, the ratio test provides no information.

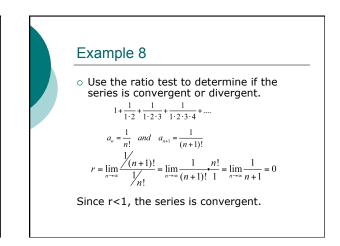
Example 7

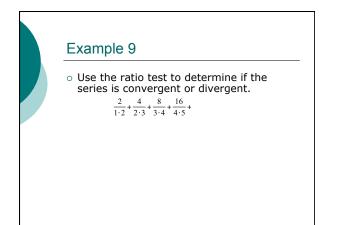
• Use the ratio test to determine if the series is convergent or divergent. $3/4 + 4/16 + 5/64 + 6/256 + \dots$

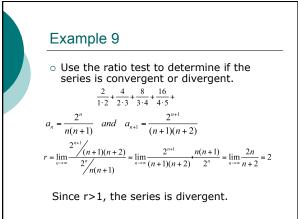


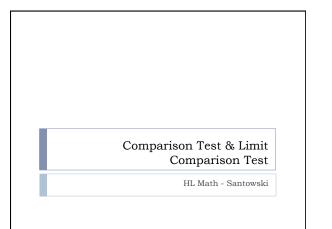
Example 8

 \circ Use the ratio test to determine if the series is convergent or divergent. $1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \dots$









Comparison test:
If the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two series with positive terms, then: (a) If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \le b_n$ for all n , then $\sum_{n=1}^{\infty} a_n$ converges. (b) If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \ge b_n$ for all n , then $\sum_{n=1}^{\infty} a_n$ diverges.
(smaller than convergent is convergent) (bigger than divergent is divergent)
Examples: $\sum_{n=2}^{\infty} \frac{3n}{n^2 - 2} > \sum_{n=2}^{\infty} \frac{3n}{n^2} = 3 \sum_{n=2}^{\infty} \frac{1}{n}$ which is a divergent harmonic series. Since the original series is larger by comparison, it is divergent.
$\sum_{n=1}^{\infty} \frac{5n}{2n^3 + n^2 + 1} < \sum_{n=1}^{\infty} \frac{5n}{2n^3} = \frac{5}{2} \sum_{n=1}^{\infty} \frac{1}{n^2}$ which is a convergent <i>p</i> -series. Since the original series is smaller by comparison, it is convergent.

