Lesson 59 – Linear Differential Equations

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Lesson Objectives

- Review the previous types of First Order Diff Eqns that we already know how to solve

- Introduce linear Diff Eqns and then solving using an "integration factor"

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(A) Linear Diff Eqns

- Consider the following Diff Eqns. Two are linear Diff Eqns and two are not

$$(a)\ \frac{dy}{dx} + 2xy = x^2$$

$$(b)\ \frac{dy}{dx} + 2xy^2 = \tan x$$

$$(c)\ \frac{dy}{dx} - \frac{1}{x}y = \cos x$$

$$(d)\ \frac{dy}{dx} + x\sqrt{y} = 2x$$

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(A) Linear ODEs

A differential equation is linear if
1. **dependent variable and its derivatives are of degree one,**
2. **coefficients of a term does not depend upon dependent variable.**

A linear first order equation is an equation that can be expressed in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Where P and Q are functions of x

(B) Example to Discuss

- Solve the linear Diff Eqn

$$\frac{dy}{dx} + 3y = e^{-x}$$

- Here is our graphical solution (slope field diagram)

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5

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(B) Example to Discuss

- Solve the linear Diff Eqn $\quad \dfrac{dy}{dx} + 3y = e^{-x}$

- <u>Multiply through</u> by e^{3x}
- $\qquad$?? $\qquad$??

- So we get $\qquad e^{3x}\dfrac{dy}{dx} + 3e^{3x}y = e^{-x} \times e^{3x}$

$$\underbrace{\qquad}_{NOTICE\,.....??}$$

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6

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(B) Example to Discuss

- Solve the linear Diff Eqn

$$\frac{dy}{dx} + 3y = e^{-x}$$

- <u>Multiply through</u> by e^{3x}
  .... So we get

$$e^{3x}\frac{dy}{dx} + 3e^{3x}y = e^{-x} \times e^{3x}$$

$$\frac{d}{dx}\left(ye^{3x}\right) = e^{2x}$$

$$\int \frac{d}{dx}\left(ye^{3x}\right) = \int e^{2x}$$

$$ye^{3x} + k = \frac{1}{2}e^{2x} + c$$

$$ye^{3x} = \frac{1}{2}e^{2x} + C$$

$$y = \frac{1}{2}e^{-x} + Ce^{-3x}$$

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7

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(B) Example to Discuss

$$y = \frac{1}{2}e^{-x} + \frac{1}{2}e^{-3x}$$

$$y = \frac{1}{2}e^{-x} - \frac{1}{2000}e^{-3x}$$

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8

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(B) Example to Discuss

- Solve the linear Diff Eqn $\dfrac{dy}{dx} + 3y = e^{-x}$

- <u>Multiply through</u> by e^{3x}

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- So HOW did we get e^{3x} as the "integrating factor"?

- We use $\quad IF = e^{\int P(x)dx}$

- (To see WHY, read through <u>Paul Dawkins notes</u> …. Link also at end of ppt)

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(C) Examples for Practice

- Solve the following linear Diff Eqns …..

$$(a)\,\frac{dv}{dt} = 9.8 - 0.196v$$

$$(b)\,\frac{dy}{dx} = -4y$$

$$(c)\,\frac{dy}{dx} - 2y = x$$

$$(d)\,\frac{dy}{dx} + 3x^2 y = x^5$$

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(D) Further Examples for Practice

- Solve the following linear Diff Eqns …..

$$(a)\,\frac{1}{x}y' - \frac{2}{x^2}y = x\cos x \quad \text{for } x > 0$$

$$(b)\,\frac{dx}{dt} - x\cos(t) = \cos(t)$$

$$(c)\ y' + y\tan(x) = \cos(x^2)\ \text{ and }\ y(0) = 2$$

$$(d)\,\frac{dy}{dx} = \cos^2(x) - y\sec(x)$$

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Further Practice ..

- http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/ordinary-differential-equations-integrating-factor.pdf

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(G) Resources

- Detailed Notes & explanations/derivations from Paul Dawkins
- http://tutorial.math.lamar.edu/Classes/DE/Linear.aspx

- From patrickJMT:
- https://www.youtube.com/watch?v=Et4Y41ZNyao
- https://www.youtube.com/watch?v=RnYzatmp-_s

- From Mathispower4u
- https://www.youtube.com/watch?v=HAb9JbBD2ig
- https://www.youtube.com/watch?v=zN0TmKEXFh8

13   Calculus - Santowski

8/20/15