Lesson 51 – Volumes of Revolution

Calculus - Santowski

Lesson Ojectives

- 1. Determine the volume of revolution of an object rotated about the x-axis
- 2. Determine by slicing (disk and washer method) or cylindrical shells to calculate volumes of solids
- 3. Apply volumes and average values to a real world problems

Fast Five – Investigation

1. Determine the area of a circle if the radius is 6 cm.
2. Determine the area of a circle if its radius is defined by y = 2x at the point where x = 3.
3. Draw the function f(x) = 2x on the interval [0,3]. Estimate the area under f(x) on [0,3] using RRAM and 3 rectangles. Draw a diagram
4. Explain what happens when each of the 3 rectangles is completely rotated around the x-axis. Draw a diagram.
5. Explain what the idea of "volume of revolution" means

(A) Volumes of Revolution

- Go to the following link and watch the animation showing the rotation of a graph about the x-axis and explaining how to determine the volume of the solid obtained in the animation above.
- http://archives.math.utk.edu/visual.calculus/5/volumes.5/index.html and go to the fifth link
- Explain the following formula to me:

$$V_x = \int_a^b A(x)dx = \int_a^b \pi \times (f(x))^2 dx = \pi \int_a^b (f(x))^2 dx$$
$$V_y = \int_a^b A(y)dy = \int_a^b \pi \times (f(y))^2 dy = \pi \int_a^b (f(y))^2 dy$$

(C) Example 1

- Example 1: Determine the volume of the solid obtained by rotating the region bounded by $f(x) = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.
- ANS: $78\pi/5$
- http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx

(D) Example 2

- (b) Example 2: Area bounded by the graphs of $f(x) = x^3 - x + 1$, $x = -1$, $x = 1$ and the x-axis.
- ANS: $226\pi/105$
- http://archives.math.utk.edu/visual.calculus/5/volumes.5/index.html

(E) Example 3

- Determine the volume of the solid formed when $y = x^2$ is rotated around the y-axis between $y = 0$ and $y = 9$

(F) Volumes of Revolution – Rings & 2 Curves

- AREA of a RING ➔ a region bounded by 2 curves
- $$Area = \pi \times (\text{outer})^2 - \pi(\text{inner})^2 = \pi\left((\text{outer})^2 - (\text{inner})^2\right)$$
- Formula to use: ➔ see animation on http://archives.math.utk.edu/visual.calculus/5/volumes.5/index.html (ring)
- $$V_x = \int_a^b A(x)dx = \int_a^b \pi \times \left[(f(x))^2 - (g(x))^2\right]dx = \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$$
  $$V_y = \int_a^b A(y)dy = \int_a^b \pi \times \left[(f(y))^2 - (g(y))^2\right]dy = \pi\int_a^b \left[(f(y))^2 - (g(y))^2\right]dy$$

(G) Example 1

- (c) Example 1: Determine the volume of the solid obtained by rotating the portion of the region bounded by the following 2 curves that lies in the first quadrant about the x-axis.

$$y = \sqrt[3]{x} \quad \text{and} \quad y = \frac{x}{4}$$

- ANS: 128π/15

- http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx

(H) Example 2

- Find the volume of the solid obtained by rotating the area bounded by $f(x) = x^2$ and $g(x) = x$ about the line $y = 2$.

- ANS: 8π/15

- http://archives.math.utk.edu/visual.calculus/5/volumes.5/index.html

(I) Example 3

- Determine the volume of the solid obtained by rotating the region bounded by the functions $y = x$ and $y = x^2 - 2x$ about the line $y = 4$.

- ANS: 153π/5

- http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx

(J) Example 4

- Determine the volume of the solid obtained by rotating the region bounded by $y = x - 1$ and $y = 2\sqrt{x-1}$ and about the line $x = -1$.

- ANS: 96π/5

- http://tutorial.math.lamar.edu/Classes/CalcI/VolumeWithRings.aspx