Warm-up

1. Find the derivative of the following function:

$$f(x) = 3x^3 - \frac{4}{x^2} + \frac{5}{x^5}$$

2. Use your answer to Question #1 to find f'(-1).

3. Use your answer to Question #2 to find the equation of the line tangent to f(x) at x = -1.

4. Write down the derivative of the following functions:

a. $g(x) = e^x$ b. $h(x) = \ln(x)$

A bit more on e^x and ln(x)...

Find the derivative of each function.

a $f(x) = 3e^x$ **b** $f(x) = x^2 + \ln x$ **c** $f(x) = \ln e^{3x}$

Write an equation for each line in questions 7–10.

7 The line tangent to the curve $f(x) = 4e^x - 7$ at $x = \ln 3$

8 The normal line to the curve $f(x) = \ln\left(e^{x^2}\right)$ at the point $(-3, 9)$

9 The line tangent to the curve $f(x) = \ln x$ at $x = e$

10 The line normal to the curve $f(x) = 2x^2 + e^{\ln x} - 3$ at $x = 2$

Show that the equation of the tangent to $y = \ln x$ at the point where $y = -1$ is $y = ex - 2$.

What happens to derivatives when we take the product of two functions?

Let $f(x) = x^5$ and $g(x) = x^7$.

1. Find $f'(x)$ and $g'(x)$

2. Find $f'(x) * g'(x)$

3. Find and simplify $f(x) * g(x)$

4. Find the derivative of the function from #3.

5. How does your answer in #2 compare to #4?

What can you conclude about the derivatives of products?

What happens to derivatives when we take the quotient of two functions?

Let $f(x) = x^5$ and $g(x) = x^7$.

1. Find $f'(x)$ and $g'(x)$

2. Find $f'(x) / g'(x)$

3. Find and simplify $f(x) / g(x)$

4. Find the derivative of the function from #3.

5. How does your answer in #2 compare to #4?

What can you conclude about the derivatives of quotients?

Try to GENERALIZE a product and/or quotient rule using function notation $f(x)$, $g(x)$, $f'(x)$, $g'(x)$

THE Product Rule

$h(x) = f(x) * g(x)$

$h'(x) =$

"One dee Two plus Two dee One"

-OR-

To the tune of Happy Birthday:

One prime two plus two prime one,

One prime two plus two prime one,

This makes up the product rule,

Golly, CALC-U-LUS is fun!

THE Quotient Rule

$h(x) = f(x) / g(x)$

$h'(x) =$

"Low dee High less High dee Low,
over Low Low"

-OR-

To the tune of Happy Birthday:

The quotient rule I need to know,

Low d high less high d low,

Draw a line then there below,

Put the squa-are of the low!

Examples to try!

1. $f(x) = 3x^2 + 4x + 5$ and $g(x) = 2x + 1$; $h(x) = f(x) * g(x)$

Find $h'(x)$

2. Find $f'(x)$ if $f(x) = x^4 \ln(x)$

3. Find $\frac{dy}{dx}$ if $y = (x^4 + x^3 + 2)(x^2 + 3x)$ *DON'T expand!

Examples to try!

1. $y = \dfrac{x^4 + 2x^3 + 2}{x^2 + 5x}$ Find $\dfrac{dy}{dx}$

2. Find $\dfrac{d}{dx}[f(x)]$ if $f(x) = \dfrac{\ln x}{x^2 + 5x + 2}$

Mixed Practice!

a $y = \dfrac{1+3x}{x^2+1}$

b $f(x) = 2x(x+1)$

d $f(x) = \dfrac{x+2}{2e^x - 3}$

a xe^x

c $y = x^3 \ln x$

g $y = e^x \ln x$

b $y = \dfrac{\sqrt{x}}{(1-2x)^2}$

c $y = x^2(2x-1)$

h. $f(x) = (3x+1)(\ln x)$

1. Suppose $y = -2x^2(x+4)$. For what values of x does $\frac{dy}{dx} = 10$?

2. If $y = \frac{2\sqrt{x}}{1-x}$, show that $\frac{dy}{dx} = \frac{x+1}{\sqrt{x}(1-x)^2}$.

3. Find the gradient of the tangent to $y = x \ln x$ at the point where $x = e$.

$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}(1-x)^2}$. For what values of x is $\frac{dy}{dx}$ i zero ii undefined?

Find the derivative of each function in questions 1 to 8.

1 $f(x) = \dfrac{x^2}{x-4}$

2 $f(x) = (2x^3 + x^2 + x)(x^2 + 1)$

3 $f(x) = \dfrac{\ln x}{x}$

4 $f(x) = e^x \ln x$

5 $f(x) = \dfrac{x-2}{x+4}$

6 $f(x) = \dfrac{e^x}{e^x + 1}$

7 $f(x) = e^x (5x^3 + 4x)$

8 $f(x) = \dfrac{2-x^2}{x^3+1}$

EXAM-STYLE QUESTIONS

9 The function $f(x) = xe^x$ has a horizontal tangent line at $x = k$
Find k.

10 Write the equations for the tangent lines to the graph of $f(x) = \dfrac{x+1}{x-1}$ that are parallel to the line $x + 2y = 10$

Extra Practice

Product Rule Practice WS

Quotient Rule Practice WS

AP Quotient and Product Rule WS

Calculus by Foerster

4-2, pg 135 #1-19 (odds only), 27

4-3, pg 139: #1-15 (odds only)