A. Lesson Context

<table>
<tr><td>BIG PICTURE of this UNIT:</td><td colspan="3">What mathematical methods are available for determining the rate at which a function changes?
Can we apply the various rules & methods associated with determining the instantaneous rate of change of a function at a point?</td></tr>
<tr><td>CONTEXT of this LESSON:</td><td>Where we’ve been

We have studied FUNCTIONS in SEM1, but from a STATIC perspective</td><td>Where we are

How do you find the average & then predict the instantaneous rates of change of a function?</td><td>Where we are heading

A generalized method for determining instantaneous rates of change</td></tr>
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B. Average Rates of Change – Graphical Perspective

average rate of change between:

(A) Jan 2013 & Jan 2014

(B) Jul 2013 & Jan 2014

(C) Sept 2013 & Jan 2014

(D) Nov 2013 & Jan 2014

(E) Dec 2013 & Jan 2014

Predict instantaneous rate of change in Jan 2014

average rate of change between:

(A) Feb ‘14 & Jan ‘15

(B) Jul ‘14 & Jan ‘15

(C) Oct ‘14 & Jan ‘15

(D) Jan 01, 2015 & Jan 17, 2015

Predict instantaneous rate of change on Jan 17, 2015

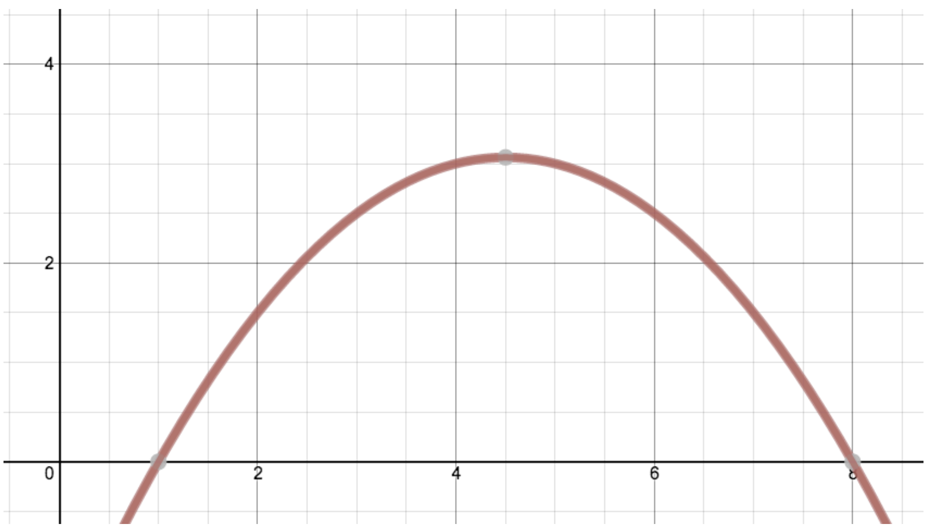
Determine the average rate of change between:

(i) t = 6 and t = 12

(ii) t = 9 and t = 12

(iii) t = 11 and t = 12

Instantaneous rate of change at t = 12



Average rate of change at:

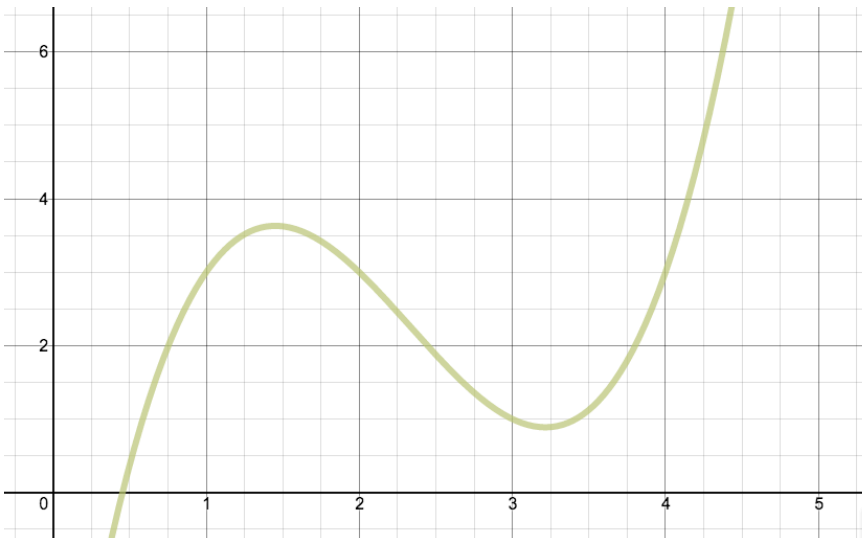
(i) t = 2 and t = 6

(ii) t = 4.5 and t = 6

(iii) t = 5 and t = 6

(iv) t = 5.5 and t = 6

Instantaneous rate of change at t = 6



Average rate of change at:

(i) t = 2 and t = 4

(ii) t = 3 and t = 4

(iii) t = 3.5 and t = 4

(iv) t = 3.75 and t = 4

Instantaneous rate of change at t = 4

C. <u>Average Rates of Change – Algebraic Perspective</u>

(A) The displacement, in meters, of a particle moving in a straight line (linear path) is modeled by $s(t) = t^2 + 2t$, where t is measured in seconds. Find the:

Average velocity over the interval:

(i) 2 – 3 seconds

(ii) 2.5 – 3 seconds

(iii) 2.9 – 3 seconds

(iv) (3 – h) to 3 seconds

Instantaneous Rate of Change at t = 3 seconds

Average rate of change over the interval:

(i) 3 – 4 seconds

(ii) 3 – 3.5 seconds

(iii) 3 – 3.1 seconds

(iv) 3 to (3 + h) seconds

Instantaneous Rate of Change at t = 3 seconds

(B) The displacement, in meters, of a particle moving in a straight line (linear path) is modeled by $x(t) = t + \frac{1}{4}t^2$, where t is measured in seconds. Find the:

Average rate of change over the interval:

(i) 0 – 1 seconds

(ii) 0.5 – 1 seconds

(iii) 0.9 – 1 seconds

(iv) (1 – h) to 1 seconds

Instantaneous Rate of Change at t = 1

Average rate of change over the interval:

(i) 1 – 2 seconds

(ii) 1 – 1.5 seconds

(iii) 1 – 1.1 seconds

(iv) 1 to (1 + h) seconds

Instantaneous Rate of Change at t = 1

(C) The displacement, in meters, of a particle moving in a straight line is modeled by $s(t) = t(t + 2)(t - 2)$, where t is measured in seconds. Find the:

Average rate of change over the interval:

(i) 0 – 1 seconds

(ii) 0.5 – 1 seconds

(iii) 0.9 – 1 seconds

(iv) $(1 - h)$ to 1 seconds

Instantaneous Rate of Change at $t = 1$

Average rate of change over the interval:

(i) 1 – 2 seconds

(ii) 1 – 1.5 seconds

(iii) 1 – 1.1 seconds

(iv) 1 to $(1 + h)$ seconds

Instantaneous Rate of Change at $t = 1$

(D) The displacement, in meters, of a particle moving in a straight line (linear path) is modeled by $x(t) = \sin(t)$, where t is measured in seconds. Find the:

Average rate of change over the interval:

(i) $\pi/2 - \pi$ seconds

(ii) $3\pi/4 - \pi$ seconds

(iii) $0.9\pi - \pi$ seconds

(iv) $(\pi - h)$ to π seconds

Instantaneous Rate of Change at $t = \pi$ s

Average rate of change over the interval:

(i) $\pi - 2\pi$ seconds

(ii) $\pi - 1.5\pi$ seconds

(iii) $\pi - 1.1\pi$ seconds

(iv) π to $(\pi + h)$ seconds

Instantaneous Rate of Change at $t = \pi$

(E) A spherical balloon is being inflated. Estimate the instantaneous rate of change of volume with respect to the radius at the instant when $r = 10$ cm.

1. Determine the average rate of change of the function $f(x)=2^x$ over the interval $2 \le x \le 4$.

2. Determine the average rate of change of the function $f(x)=2^x$ over the interval $3 \le x \le 4$.

3. Determine the average rate of change of the function $f(x)=2^x$ over the interval $3.9 \le x \le 4$.

4. Determine the average rate of change of the function $f(x)=2^x$ over the interval $3.999 \le x \le 4$.

5. Use your answers from questions 1 to 4 to estimate the instantaneous rate of change of the function $f(x)=2^x$ at $x=4$ to two decimal places.

6. If the $f(x)=2^x$ represents the number of bacteria present at time x in hours. What are the units of the rate of change?

7. Determine the instantaneous rate of change for each function at the indicated point:
   a) $f(x)=\frac{1}{x}; x=5$
   b) $g(x)=2x^4-5x^3; x=1$
   c) $p(x)=3\cos x-2; x=3$

8. Use the algebraic method, difference quotient, $I.R.O.C. \square \frac{f(x+h)-f(x)}{h}$, determine the slope of the tangent line to $f(x)=-x^2+5x-3$ at $x=2$.
9. Use your answer from #8 to determine the equation of the tangent line at $x=2$.

EXAMPLE 18.4

The concentration of a drug, in milligrams per millilitre, in a patient's bloodstream, t hours after an injection is approximately modelled by the function

$$t \mapsto \frac{2t}{8+t^3}, t \ge 0.$$

Find the average rate of change in the concentration of the drug present in a patient's bloodstream;
(a) during the first hour.
(b) during the first two hours.
(c) during the period $t = 2$ to $t = 4$.

GENERALIZATION ➔ INSTANTANEOUS RATE OF CHANGE AT x = a GIVEN THE FUNCTION y = f(x) ➔