Lesson 29 – Circle Geometry – Arcs & Sectors

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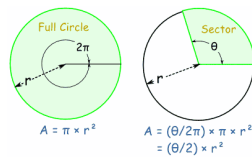1/11/15

(A) Area of a Sector

- Recall the area of a circle formula: $A = \pi r^2$
- So how would you find the area of:
- (i) half a circle
- (ii) a quarter of a circle
- (iii) one tenth of a circle
- (iv) the area swept out by the terminal arm as it rotates 36 degrees?
- (v) the area swept out by the terminal arm as it rotates 90 degrees?
- (vi) the area swept out by the terminal arm as it rotates $\pi/2$ radians?

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(F) Area of a Sector

- CONCLUSION from previous slide ➔ You can work out the Area of a Sector by comparing its angle to the angle of a full circle.
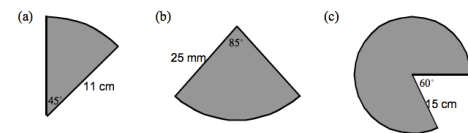- Note: I am using underline radians for the angles.

Full Circle
2π
$A = \pi \times r^2$

Sector
θ
r
$A = (\theta/2\pi) \times \pi \times r^2$
$= (\theta/2) \times r^2$

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Sectors - Examples - Basics

2. Calculate the area of each sector below.

(a) 45° 11 cm

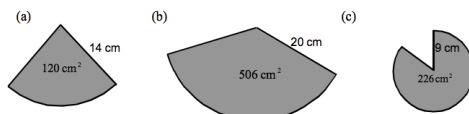(b) 25 mm 85°

(c) 60° 5 cm

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4

Sectors - Examples - Basics

4. In each diagram below the area of the sector is given. Calculate the size of the angle at the centre of the sector.

(a) 14 cm 120 cm²

(b) 20 cm 506 cm²

(c) 9 cm 226 cm²

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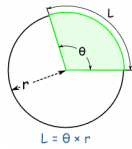
1/11/15

5

(G) Arc Length

- Recall the circumference of a circle formula: $A = 2\pi r$
- So how would you find the arc length of:
- (i) half a circle
- (ii) a quarter of a circle
- (iii) one tenth of a circle
- (iv) the arc length traveled by the terminal arm as it rotates 36 degrees?
- (v) the arc length traveled by the terminal arm as it rotates 90 degrees?
- (vi) the arc length traveled by the terminal arm as it rotates $\pi/2$ radians?

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(G) Arc Length

- Arc Length of Sector or Segment
- By the same reasoning, the arc length (of a Sector or Segment) is Arc Length "L" = $\theta \times r$
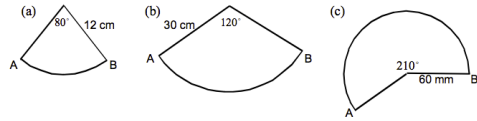- = $(\theta \times \pi/180) \times r$ *(if θ is in degrees)*
-

$$L = \theta \times r$$

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Arc Length - Examples - Basics

1. Calculate the length of arc AB in each question below
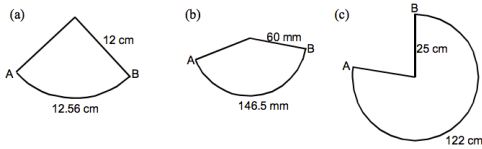
(a) 80° 12 cm

(b) 30 cm 120°

(c) 210° 60 mm

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Arc Length - Examples - Basics

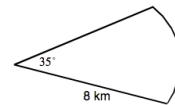3. In each diagram below the length of arc AB is given. Calculate the size of the angle at the centre of the sector.

(a) 12 cm 12.56 cm

(b) 60 mm 146.5 mm

(c) 25 cm 122 cm

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Examples - Applications

5. The beam from a lighthouse reaches a distance of 8 kilometres and spreads to an angle of 35°.

35° 8 km

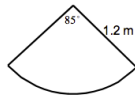Calculate the area covered by the beam from the lighthouse.

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Examples - Applications

6. The curved part on an anchor is in the shape of an arc of a circle which has radius 1.2 metres.
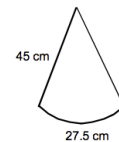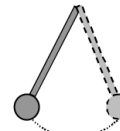
85° 1.2 m

Calculate the length of this arc.

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Examples - Applications

7. A pendulum is 45 centimetres long. When the pendulum swings it travels along the arc of a circle and covers a distance of 27.5 centimetres.

45 cm 27.5 cm

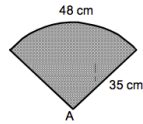Calculate the size of the angle through which the pendulum travels.

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1/11/15

Examples - Applications

10. A fan is in the shape of an arc of a circle with radius 35 centimetres.

48 cm

35 cm

A

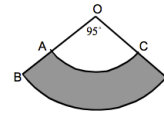Calculate the size of the angle at A.

Examples - Applications

13. In the diagram below AC and BD are arcs of circles with centres at O. The radius, OA, is 10 centimetres and the radius, OB, is 16 centimetres.
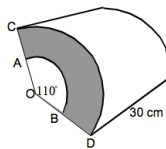
O
95°
A   C
B   D

Find the shaded area.

Examples - Applications

14. The diagram shows a prism whose cross-section is the area between two sectors. One sector has radius OA = 12 centimetres and the other has radius OC = 15 centimetres. Calculate the volume of this prism.
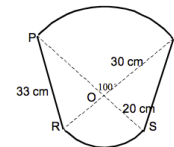
C
A
O 110°
B
D
30 cm

Examples - Applications

15. In the diagram PQ and RS are arcs of circles with centre O. The radius, OQ, is 30 centimetres long and the radius, OS, is 20 centimetres long.

P   Q
30 cm
33 cm   100°
O
20 cm
R   S

Calculate the perimeter of the shape.
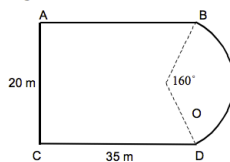
Examples - Applications

16. The diagram below shows an ornamental garden. The garden is in the shape of a rectangle with a sector of a circle added at one end. The length of the garden is 35 metres and its breadth is 20 metres.

A   B

20 m   160°
O

C   35 m   D
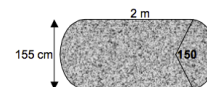
(a) Calculate OB the radius of the sector.
(b) Find the perimeter of the garden.

Examples - Applications

17. A worktop is in the shape of a rectangle with identical sectors of a circle, centre O, at each end. The width of the tabletop is 155 centimetres and its length is 2 metres.

2 m

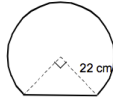155 cm   150

Calculate the perimeter of the worktop.

Examples - Applications

18. The diagram below shows a mirror. The mirror is in the shape of the sector of a circle with a straight base. The radius of the sector is 22 centimetres.

22 cm

Calculate the distance round the outside of the mirror.