

Lesson 26 – Quadratic Trigonometric Equations

IB Math HL - Santowski

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FAST FIVE

Solve each equation by factoring

- Solve the quadratic equation $x^2 + 2x = 15$ algebraically
 1. $(k+1)(k-5) = 0$
 2. $(a+1)(a+2) = 0$
 3. $(4k+5)(k+1) = 0$
 4. $(2m+3)(4m+3) = 0$
- Solve the quadratic equation $x^2 + 2x = 15$ graphically
 5. $x^2 - 11x + 19 = -5$
 6. $n^2 + 7n + 15 = 5$
 7. $n^2 - 10n + 22 = -2$
 8. $n^2 + 3n - 12 = 6$
 9. $6n^2 - 18n - 18 = 6$
 10. $7r^2 - 14r = -7$

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(A) Prerequisite Skill: Factoring with Trig

- Factor the following trig expressions:

Factor.

- | | |
|---|---|
| (a) $\sin^2 \theta - \sin \theta$ | (b) $\cos^2 \theta - 2 \cos \theta + 1$ |
| (c) $3 \sin^2 \theta - \sin \theta - 2$ | (d) $4 \cos^2 \theta - 1$ |

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(B) Solving Quadratic Trigonometric Equations

- We will outline a process by which we come up with the solution to a trigonometric equation ANALYTICALLY using ALGEBRA
- We will outline a process by which we come up with the solution to a trigonometric equation GRAPHICALLY using ALGEBRA
- → it is important you understand WHY we carry out these steps, rather than simply memorizing them and simply repeating them on a test or quiz

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(B) Solving Quadratic Trigonometric Equations

4. Solve each equation for x , $0^\circ \leq x \leq 360^\circ$.
- (a) $\sin x \cos x = 0$
 - (b) $\sin x (\cos x - 1) = 0$
 - (c) $(\sin x + 1) \cos x = 0$
 - (d) $\cos x (2 \sin x - \sqrt{3}) = 0$
 - (e) $(\sqrt{2} \sin x - 1)(\sqrt{2} \sin x + 1) = 0$
 - (f) $(\sin x - 1)(\cos x + 1) = 0$

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(B) Solving Quadratic Trigonometric Equations

- Solve $\sin^2 x - 1 = 0$ on the domain $0 \leq x \leq 4\pi$.
- Your first solution will be analytical.
- We will VERIFY with a graphic solution

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(B) Solving Quadratic Trigonometric Equations

- Solve the equation $2\cos^2x - 1 = -\cosx$ on the domain $0 \leq x \leq 4\pi$.
- Your first solution will be analytical.
- We will VERIFY with a graphic solution

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(B) Solving Quadratic Trigonometric Equations

- Solve the equation $8\sin^2x + 13\sinx = 4 - 4\sin^2x$ on the domain $-\pi \leq x \leq 3\pi$.
- Your first solution will be analytical.
- We will VERIFY with a graphic solution

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(\theta) = 1$ if $0^\circ \leq \theta \leq 360^\circ$

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(\theta) = 1$ if $0^\circ \leq \theta \leq 360^\circ$

$$2\cos^2(\theta) = 1$$

$$\cos^2(\theta) = \frac{1}{2}$$

$$\cos(\theta) = \pm \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(\theta - 45^\circ) = 1$ if $0^\circ \leq \theta \leq 360^\circ$

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(\theta + \pi/3) = 1$ if $-2\pi \leq \theta \leq 5\pi$

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(B) Solving Quadratic Trigonometric Equations

- Solve $\cos^2(x) + 2\cos(x) = 0$ for $0 \leq x \leq 2\pi$

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(B) Solving Quadratic Trigonometric Equations

- Solve $\cos^2(x) + 2\cos(x) = 0$ for $0 \leq x \leq 2\pi$

$$\begin{aligned}\cos^2(x) + 2\cos(x) &= 0 \\ \cos(x) \times (\cos(x) + 2) &= 0 \\ (i) \therefore \cos(x) &= 0 \\ x &= \cos^{-1}(0) \\ x &= \frac{\pi}{2}, \frac{3\pi}{2} \\ (ii) \therefore \cos(x) &= -2 \\ x &= \cos^{-1}(-2) \\ x &\notin \mathbb{R}\end{aligned}$$

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(B) Solving Quadratic Trigonometric Equations

- Solve $\cos^2(2x) + 2\cos(2x) = 0$ for $0 \leq x \leq 2\pi$

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(x) - 3\cos(x) + 1 = 0$ for $0 \leq x \leq 2\pi$

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(x) - 3\cos(x) + 1 = 0$ for $0 \leq x \leq 2\pi$

$$\begin{aligned}2\cos^2(x) - 3\cos(x) + 1 &= 0 \\ (2\cos(x) - 1)(\cos(x) - 1) &= 0 \\ (i) \therefore 2\cos(x) - 1 &= 0 \\ \cos(x) &= \frac{1}{2} \\ x &= \frac{\pi}{3}, \frac{5\pi}{3} \\ (ii) \therefore \cos(x) &= 1 \\ x &= 0, 2\pi\end{aligned}$$

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(B) Solving Quadratic Trigonometric Equations

- Solve $2\cos^2(2x - 2\pi/3) - 3\cos(2x - 2\pi/3) + 1 = 0$ for $x \in \mathbb{R}$

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(C) Further Examples

- Solve the following without a calculator

Solve each equation for x , $0 \leq x \leq 2\pi$.

- (a) $(2 \sin x - 1) \cos x = 0$
- (b) $(\sin x + 1)^2 = 0$
- (c) $(2 \cos x + \sqrt{3}) \sin x = 0$
- (d) $(2 \cos x - 1)(2 \sin x + \sqrt{3}) = 0$
- (e) $(\sqrt{2} \cos x - 1)(\sqrt{2} \cos x + 1) = 0$
- (f) $(\sin x + 1)(\cos x - 1) = 0$

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(C) Further Examples

- Solve the following algebraically, without a GRAPHICAL approach

Solve for θ to the nearest hundredth of a radian, $0 \leq \theta \leq 2\pi$.

- (a) $2 \cos^2 \theta + \cos \theta - 1 = 0$
- (b) $2 \sin^2 \theta = 1 - \sin \theta$
- (c) $\cos^2 \theta = 2 + \cos \theta$
- (d) $2 \sin^2 \theta + 5 \sin \theta - 3 = 0$
- (e) $3 \tan^2 \theta - 2 \tan \theta = 1$
- (f) $12 \sin^2 \theta + \sin \theta - 6 = 0$

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(D) Homework

- Nelson Textbook, Chap 6.6
- <http://mrsantowski.tripod.com/2010MathSLY1/Assessments/NelsonS66p541.pdf>

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