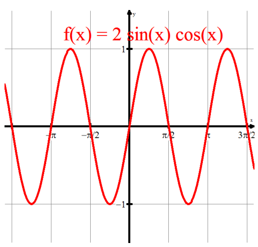
Lesson 24 – Double Angle & Half Angle Identities

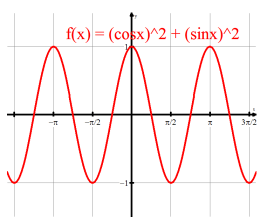
HL Math – Santowski

Fast Five

- Graph the following functions on your TI-84 and develop an alternative equation for the graphed function (i.e. Develop an identity for the given functions)
- f(x) = 2sin(x)cos(x)
- g(x) = $\cos^2(x) - \sin^2(x)$

Fast Five





(A) Review

- List the six new identities that we call the addition subtraction identities

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$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

(B) Using the Addition/Subtraction Identities

- We can use the addition/subtraction identities to develop new identities:
- Develop a new identity for:
- (a) if sin(2x) = sin(x + x), then ….
- (b) if cos(2x) = cos(x + x), then ….
- (c) if tan(2x) = tan(x + x), then ……

(B) Double Angle Formulas

- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$

(B) Double Angle Formulas

- Working with $\cos(2x) = \cos^2(x) - \sin^2(x)$
- But recall the Pythagorean Identity where $\sin^2(x) + \cos^2(x) = 1$
- So $\sin^2(x) = 1 - \cos^2(x)$
- And $\cos^2(x) = 1 - \sin^2(x)$
- So $\cos(2x) = \cos^2(x) - (1 - \cos^2(x)) = 2\cos^2(x) - 1$
- And $\cos(2x) = (1 - \sin^2(x)) - \sin^2(x) = 1 - 2\sin^2(x)$

(B) Double Angle Formulas

- Working with cos(2x)
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(2x) = 2\cos^2(x) - 1$
- $\cos(2x) = 1 - 2\sin^2(x)$
- $\sin(2x) = 2\sin(x)\cos(x)$

(C) Double Angle Identities - Applications

- (a) Determine the value of sin(2θ) and cos(2θ) if

  $$\sin(\theta) = \frac{1}{4} \quad \text{and} \quad \frac{\pi}{2} \le \theta \le \pi$$

- (b) Determine the values of sin(2θ) and cos(2θ) if

  $$\cos(\theta) = -\frac{\sqrt{5}}{6} \quad \text{and} \quad \pi \le \theta \le \frac{3\pi}{2}$$

- (c) Determine the values of sin(2θ) and cos(2θ) if

  $$\tan(\theta) = \frac{2}{5} \quad \text{and} \quad \pi \le \theta \le \frac{3\pi}{2}$$

(C) Double Angle Identities - Applications

- Solve the following equations, making use of the double angle formulas for key substitutions:

$$\begin{aligned}
&\text{(a) } \cos 2x = \cos^2 x \quad \text{for } -\pi \le x \le \pi \\
&\text{(b) } \sin 2x = \cos x \quad \text{for } -\pi \le 2x \le \pi \\
&\text{(c) } \sin 2x + \sin x = 0 \quad \text{for } 0 \le x \le 2\pi \\
&\text{(d) } \cos 2x + \cos x = 0 \quad \text{for } 0 \le x \le 2\pi \\
&\text{(e) } \cos^2 x - 2\sin x\cos x - \sin^2 x = 0 \quad \text{for } 0 \le 2x \le \pi
\end{aligned}$$

(C) Double Angle Identities - Applications

- Write as a single function:

  (a) $\sin x \cos x$

  (b) $2\cos^2 x - 2\sin^2 x$

- State the period & amplitude of the single function

(B) Double Angle Formulas

- Working with cos(2x)

- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(2x) = 2\cos^2(x) - 1$
- $\cos(2x) = 1 - 2\sin^2(x)$

- $\sin(2x) = 2\sin(x)\cos(x)$

(C) Double Angle Identities - Applications

- (a) If sin(x) = 21/29, where 0º ≤ x ≤ 90º, evaluate: (i) sin(2x), (ii) cos(2x), (iii) tan(2x)
- (b) SOLVE the equation cos(2x) + cos(x) = 0 for 0º ≤ x ≤ 360º
- (c) Solve the equation sin(2x) + sin(x) = 0 for -180º ≤ x ≤ 540º.
- (d) Write sin(3x) in terms of sin(x)
- (e) Write cos(3x) in terms of cos(x)

(C) Double Angle Identities - Applications

- Simplify the following expressions:

$$\text{(a)}\ \frac{\sin 2x}{\cos x} \qquad \text{(b)}\ \cos(2x)+1$$
$$\text{(c)}\ \frac{\cos x \sin 2x}{1+\cos 2x} \qquad \text{(d)}\ \cos(2x)+2\sin^2(x)+1$$
$$\text{(e)}\ \frac{\cos 2x}{\cos x+\sin x} \qquad \text{(f)}\ \frac{\cos 2x}{\cos x-\sin x}-\sin x$$

(C) Double Angle Identities - Applications

- Use the double angle identities to prove that the following equations are identities:

$$\text{(a)}\ \frac{\sin 2x}{1+\cos 2x}=\tan x$$
$$\text{(b)}\ \frac{1-\sin 2x}{\cos 2x}=\frac{\cos 2x}{1+\sin 2x}$$
$$\text{(c)}\ \frac{\sin 2x}{\sin x}=4\cos x-\frac{\cos 2x+1}{\cos x}$$

(E) Half Angle Identities

- Start with the identity $\cos(2x) = 1 - 2\sin^2(x)$
- Isolate $\sin^2(x)$ ➔ this is called a "power reducing" identity ➔ Why?
- Now, make the substitution x = θ/2 and isolate sin(θ/2)

- Start with the identity $\cos(2x) = 2\cos^2(x) - 1$
- Isolate $\cos^2(x)$ ➔ this is called a "power reducing" identity ➔ Why?
- Now, make the substitution x = θ/2 and isolate cos(θ/2)

(E) Half Angle Identities

- So the new half angle identities are:

$$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\cos\theta}{2}}$$

(F) Using the Half Angle Formulas

- (a) If sin(x) = 21/29, where 0º ≤ x ≤ 90º, evaluate: (i) sin(x/2), (ii) cos(x/2)
- (b) develop a formula for tan(x/2)
- (c) Use the half-angle formulas to evaluate:

$$\text{(a) } \sin\left(\frac{\pi}{12}\right) \qquad \text{(b) } \cos\left(\frac{7\pi}{12}\right)$$
$$\text{(c) } \sin(112.5°) \qquad \text{(d) } \cos(67.5°)$$
$$\text{(e) } \sin(225°)$$

(F) Using the Half Angle Formulas

- Prove the identity:

$$\text{(a) } \sin^2\left(\frac{x}{2}\right) = \frac{\sin^2 x}{2 + 2\cos x}$$
$$\text{(b) } 2\sec(x)\cos^2\left(\frac{x}{2}\right) = 1 + \sec(x)$$

(F) Homework

- HW
- S14.5, p920-922, Q8-13,20-23, 25-39odds