(A) Lesson Context

BIG PICTURE of this UNIT:	• What are & how & why do we use exponential and logarithmic functions? • proficiency with algebraic manipulations/calculations pertinent to exponential & logarithmic functions • proficiency with graphic representations of exponential & logarithmic functions		
CONTEXT of this LESSON:	Where we've been Previous math courses working with Exponent Laws and graphs of Exponential Relations	Where we are Reviewing the exponent laws and working with rational exponents	Where we are heading Working with Exponential functions in modeling problems and as functions (transformations & inverses)

(B) Lesson Objectives

a. Review the basic Exponent Laws

(C) Exponent Laws

Definition of the terms in an exponential equation: $b^x = p$

- b is the base (of the exponent)
- x is the exponent
- p is the power (the result of repeatedly multiplying b by itself, x number of times, or a base raised to an exponent)

Example: In $2^3 = 8$, the base is 2, the exponent is 3 and the power is 8. This can be read as the following:

- "Two cubed is 8."
- "Two to the exponent 3 is 8."
- "Two to the 3 is 8."
- "Eight is the third power of 2."
- BUT it CANNOT be read as: "Two to the power 3 is 8." (The power is NOT 3 - the power is 8 and the EXPONENT is 3!)

EXPONENT LAWS:

1. Comparison of bases: If two powers have the same bases, then their exponents must be equal.
   - $a^x = b^x$ if and only if $a = b$ $(x \neq 0,\ a > 0,\ b > 0)$

2. Comparison of exponents: If two powers have the same exponents, then their bases must be equal.
   - exponents (with like bases): $b^x = b^y$ if and only if $x = y$ $(b \neq -1, 0, 1)$

3. Multiplication of like bases: When multiplying (2 or more) like bases, keep the base and ADD the exponents.
   - $b^x \cdot b^y = b^{x+y}$

4. Division of like bases: When dividing like bases, keep the base and SUBTRACT the exponents.
   - $\frac{b^x}{b^y} = b^{x-y}$ (as long as $b \neq 0$)

5. Power of a product: If a single term is being raised to an exponent, then the exponent applies to each *factor* of the single term.
   - $(ab)^x = a^x b^x$
   - Common mistake: $(a+b)^x \neq a^x + b^x$ (this is NOT TRUE because the base of $(a+b)$ is not a single term, but rather two terms)

6. Power of a quotient: If a fraction is being raised to an exponent, then the exponent applies to both the numerator and the denominator of the fraction.
   - $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, b \neq 0$ $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$ (why can't b equal zero?)

7. Power of a power: When a power (such as b^x) is being raised to another (outer) exponent, the result is called a power of a power. In this case, keep the base and multiply exponents.
   - $(b^x)^y = b^{xy}$
   - Because the order of multiplication (commutativity) does not matter, these are equivalent: $(b^x)^y = b^{xy}$ and $(b^y)^x = b^{yx}$.

8. Exponent of zero: Any base raised to an exponent of zero (or the zeroeth power of any base) is ALWAYS equal to one.
   - $b^0 = 1$
   - One exception is 0^0; this is a non-unique or **indeterminate** value that arises often in calculus.

9. Negative exponent: When a base is raised to a negative exponent, reciprocate the base and raise the result to the positive exponent.
   - $b^{-x} = \frac{1}{b^x}$, $b \neq 0$ (why can't b equal zero?)

10. Fractional exponent: When the exponent of a base is a fraction, the numerator of the fractional exponent acts as a regular exponent while the denominator of the fractional exponent indicates a root of the base.
    - $b^{\frac{m}{n}} = \sqrt[n]{b^m}$
    - The symbol $\sqrt[n]{\ }$ is called a radical or n^th-root symbol. The number n in the "V" is called the index (or type of root). If no number is specified, the type of root is automatically a SQUARE root. Otherwise, refer to the root as the "n^{th} root", as in $\sqrt[8]{56}$ is the eighth root of 56.
    - You can either work out the base raised to the exponent first and then take the root: $b^{\frac{m}{n}} = \sqrt[n]{\left(b^m\right)}$ OR you can work out the n^{th} root of the base first and then apply the exponent: $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$.
    - Advanced lingo: The base of the exponential expression $\left(\sqrt[n]{b}\right)^m$ is $\left(\sqrt[n]{b}\right)$, the exponent on the base is m and the (m^{th}) power of the base is the result $\left(\sqrt[n]{b}\right)^m$. What are the base, exponent and power of $\sqrt[n]{\left(b^m\right)}$?

Exercises:

1. Identify the parts of an exponential equation. State the base, the exponent and the power for each.

a) $(-4)^3 = -64$

b) $2^{-5} = \frac{1}{32}$

c) $e^2 = p$

d) $j^0 = 1$

e) $\sqrt[3]{n} = z$

f) $\left(\sqrt[3]{n}\right)^k = y$

2. Use the exponent laws to write each expression with a single, simplified base.

a) $x^4 \cdot x^5 \cdot x^9$

b) $x^4 \cdot x^{-5}$

c) $\frac{x^{12}}{x^4}$

d) $\frac{a^{10}}{a^{14}}$

e) $\frac{a}{a^{-5}}$

f) $\left(g^7\right)^{20}$

g) $\frac{\left(k^a\right)^b \cdot k^{3ab}}{k^{7ab}}$

h) $\left(\sqrt{x}\right)^6$

3. Use the exponent laws to write each expression without any zero, negative or fractional exponents.

a) w^{-2}

b) $\frac{\left(a^2\right)^3}{a^7}$

c) $x^{\frac{4}{5}}$

d) $x^{-\frac{4}{5}}$

e) $\frac{\left(r^3\right)^{-1} \cdot r^{-5}}{\left(r^{-4}\right)^2}$

f) $\left(\frac{1}{2} + \frac{2}{3} \cdot \frac{3}{4} - \frac{4}{5} \div \frac{5}{6}\right)^0$

4. Rewrite the following expressions without a fractional exponent (where applicable) and simplify the (resulting) radicals.

a) $\sqrt{ab^2c^3d^{10}e^{21}}$

b) $\left(a^7b^6c^5d^4e^3f^2\right)^{\frac{1}{3}}$

5. Simplify the following expressions so that the final answers contain as few bases as possible but does not contain zero, negative or fractional exponents.

a) $x^5y^7z^{-10} \cdot \left(x^2y^3z^4\right)^3$

b) $\left(a^2b^3c^{-1}\right)^3 \cdot \left(\frac{c^5}{a^6b^4}\right)$

c) $\left(\frac{\sqrt[5]{m^7n^{13}p^4q^{101}}}{mn^3pq^{18}}\right)^{-2}$

d) $\left(\sqrt[6]{\frac{f^4g^{-2}h^0}{f^{-3}g}}\right)^{12}$

6. Simplify the following.

a) $x^{\frac{1}{2}}\left(x^{\frac{3}{2}}+2x^{\frac{1}{2}}\right)$

d) $\left(\left(x^{-2}y^{3}\right)^{-2}\right)^{2}$

g) $\sqrt[3]{128x^{4}y^{2}z^{9}}$

b) $\left(\dfrac{3x^{-2}y^{-4}}{y^{-3}x^{-7}}\right)^{2}$

e) $\dfrac{\left(3a^{-1}\right)^{2}}{3\left(a^{-1}\right)^{-2}}$

h) $\left(a^{-x}b^{-x}\right)^{-1}$

c) $\left(\dfrac{9x^{4}y^{4}}{x^{-2}y^{2}}\right)^{-\frac{1}{2}}$

f) $\left(\dfrac{-d^{10}}{-49b^{6}}\right)^{-0.5}$

i) $\left(a^{-x}+b^{-x}\right)^{-1}$

7. Evaluate (simplify as a number) the following.

a) -3^{2}

f) $8^{\frac{1}{3}}$

k) $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$

b) $\left(-3\right)^{2}$

g) $8^{-\frac{1}{3}}$

l) $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}}$

c) -3^{-2}

h) $8^{-\frac{4}{3}}$

m) $2^{3^{2}}$

d) $\left(-3\right)^{-2}$

i) $16^{\frac{3}{4}}$

n) $\left(100^{\frac{1}{2}}-36^{\frac{1}{2}}\right)^{2}$

e) $\left(3^{-2}+3^{-3}\right)^{-1}$

j) $16^{-0.5}$

o) $\left(\dfrac{81}{5^{4}}\right)^{0.25}$

- Factor x^{16}
- Factorize 7^8. (no calculators)
- Factorize -6^{-9}. (no calculators)
- Factorize $(x^6 - x^{-8})$

- Expand $(x^4 - y^{-2})^2$
- Expand $(x^4y^{-2})^2$

- Evaluate $(1 + 1/x)^x$ if $x = \{5, 15, 30, 90\}$

Expand and simplify

$(i)\ x^{-\frac{1}{2}}\left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\right)$

$(ii)\ (2^x + 3)(2^{x+1} + 1)$

$(iii)\ (3^x - 3^{-x})^2$

(a) Simplify $\dfrac{x^{-1} - y^{-1}}{x - y}$

(b) Is $\left(\dfrac{1}{x^{-1}} + \dfrac{1}{y^{-1}}\right)^{-1} = \dfrac{x + y}{xy}$

(c) Simplify $\dfrac{x^{-2}}{x^{-2} + y^{-2}} + \dfrac{y^{-2}}{x^{-2} - y^{-2}}$

(c) Simplify $\dfrac{x^{-1} + y^{-1}}{x^{-1}} + \dfrac{x^{-1} - y^{-1}}{y^{-1}}$

ex 1. Simplify the following expressions:

(i) $(3a^2b)(-2a^3b^2)$

(ii) $(2m^3)^4$

(iii) $(-4p^3q^2)^3$

ex 2. Simplify $(6x^5y^3/8y^4)^2$

ex 3. Simplify $(-6x^{-2}y)(-9x^{-5}y^{-2}) / (3x^2y^{-4})$ and express answer with positive exponents

ex 4. Evaluate the following

(i) $(3/4)^{-2}$

(ii) $(-6)^0 / (2^{-3})$

(iii) $(2^{-4} + 2^{-6}) / (2^{-3})$

We will use the various laws of exponents to simplify expressions.

ex. $27^{1/3}$

ex. $(-32^{0.4})$

ex. $81^{-3/4}$

ex. Evaluate $49^{1.5} + 256^{-1/4} - 27^{-2/3}$

ex. Evaluate $4^{1/2} + (-8)^{-1/3} - 27^{4/3}$

ex. Evaluate $\sqrt[3]{8} + \sqrt[4]{16} - 125^{-4/3}$

ex. Evaluate $(4/9)^{\frac{1}{2}} + (4/25)^{3/2}$