Lesson 10 – Inverses & Inverse Functions

IBHL1 Math - Santowski

Fast Five – Skills Preview

- Isolate "x" in the following equations (make "x" the subject of the equation)

$(a)\ 3x - 2y + 7 = 0$

$(b)\ f(x) = \frac{1}{2}(x+2)^2 - 5$

$(c)\ h(x) = \frac{2}{3 - x}$

$(d)\ y = x^2 - 4x + 1$

$(e)\ y = e^{x+1}$

Faster Five – Skills Preview

- For the following pairs of functions
- (a) Determine fog (x)
- (b) Determine gof (x)
- (c) Graph the original two functions in a square view window & make observations about the graph ➔ then relate these observations back to the composition result

$(a)\ f(x) = 3x - 6 \quad \text{and} \quad g(x) = \frac{1}{3}x + 2$

$(b)\ f(x) = \frac{1}{x+3} \quad \text{and} \quad g(x) = \frac{1 - 3x}{x}$

$(c)\ f(x) = 3 - (x+2)^2 \quad \text{where } x \geq -2$

$\text{and} \quad g(x) = \sqrt{3 - x} - 2$

$(d)\ f(x) = e^{2x-1} \quad \text{and} \quad g(x) = \frac{1}{2}(\ln(x) - 1)$

Lesson Objectives

- Find the inverse of a function from numeric/tabular, graphic or algebraic data
- Compose a function with its inverse to develop the identity function
- Understand inverses as transformations

The **BIG** Picture

QUESTIONS??

- Are all functions invertible?
- Do function inverses "do the same thing" as our additive/multiplicative inverses?
- Why "invert" a function in the first place?

(A) Inverses – The Concept

- Let's back to our input ➔ output notion for functions.
- If functions are nothing more than input/output operators, then the concept of an inverse has us considering **how to go in reverse** ➔ going from the output back to the input

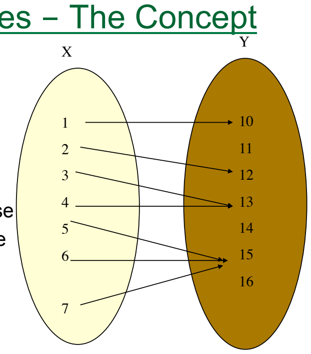
(A) Inverses – The Concept

- If the elements of the ordered pairs or mappings of a function are **reversed**, the resulting set of ordered pairs or mappings are referred to as the INVERSE.
- Since we are **REVERSING** the elements ➔another point worth noting: the domain of the original function now becomes the range of the inverse; likewise, the range of the original becomes the domain of the inverse.

(A) Inverses – The Concept

Determine:

(a) Domain of f(x)
(b) Range of f(x)
(c) Mapping of inverse
(d) Domain of inverse
(e) Range of inverse
(f) Is the inverse a function?



(B) Notation of the Inverses

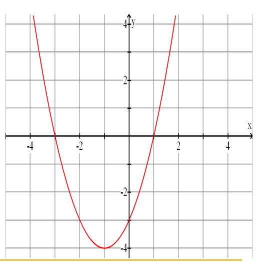
- If the inverse relation IS a function, then the notation used for these inverses is $f^{-1}(x)$.
- IMPORTANT NOTE:
- $f^{-1}(x)$ does not mean $(f(x))^{-1}$ or $1 / f(x)$.

(C) Examples

- Determine the equation for the inverse of the following functions. Draw both graphs and find the D and R of each.
- (1) $y = 4x - 9$
- (2) $y = 2x^2 + 4$ but …..
- (3) $y = 2 - \sqrt{x+3}$
- And some rational functions
- (5) $y = \dfrac{2}{3x-9}$
- (6) $y = \dfrac{2x-1}{3x-9}$
- (7) $y = |x|$

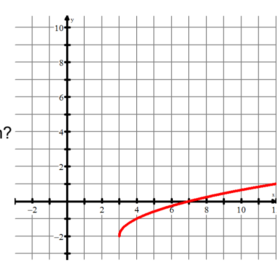
(C) Examples

- Consider a graph of the following data:
- 1. State domain and range of *f*
- 2. Evaluate f(-2), f(0),
- 3. Graph the inverse relation
- 4. Is the inverse a function?
- 5. **HOW can we make the inverse a function?**
- 6. Evaluate $f^{-1}(1)$, $f^{-1}(-2)$
- 7. State the domain and range of $f^{-1}(x)$
- Here is the graph of f(x)



(C) Examples

- Given the function of y = f(x), determine
- (a) the domain and range of the relation
- (b) the domain and range of the inverse relation
- (c) Is the inverse relation a function?
- (d) Evaluate $f \circ f^{-1}(8)$
- (e) Evaluate f(4)
- (f) Solve f(x) = ½
- (g) Evaluate $f^{-1}(1)$
- (h) Solve $f^{-1}(x) = 7$



(C) Examples

- Given the function of , determine $f(x) = 1 - \sqrt{x-2}$
- (a) the domain and range of the relation
- (b) the domain and range of the inverse relation
- (c) Determine the equation of the inverse relation
- (d) Is the inverse relation a function?
- (e) Evaluate $fof^{-1}(5)$
- (f) Evaluate f(6)
- (g) Solve f(x) = -2
- (h) Evaluate $f^{-1}(-3)$
- (i) Solve $f^{-1}(x) = 11$

(D) Existence of Inverses

- A function, *f*, has an inverse, f^{-1}, if and only if *f* is one to one.
- So how do we "test" if a function is one to one?
- Let a be any real number, so if x = a, then y = f(a)
- Let b be any real number, so if x = b, then y = f(b)
- Recall that is f(x) is one to one, then one output is produced by one input AND ALSO one input produces one output
- So, if our two outputs (f(a) & f(b)) are equal AND if our two inputs are NOT the same, then our function is NOT one to one

(D) Existence of Inverses

- Let f(x) = $2+\sqrt{x}$
- So f(a) = $2 + \sqrt{a}$
- And f(b) = $2 + \sqrt{b}$
- If f(a) = f(b)
- Then $2 + \sqrt{a} = 2 + \sqrt{b}$
- So $\sqrt{a}=\sqrt{b}$
- Meaning that a = b
- so the only way to get the two same output values is to have the two input values to be identical, meaning you MUST be one to one.

- Let f(x) = $2 + x^2$
- So f(a) = $2 + a^2$
- And f(b) = $2 + b^2$
- If f(a) = f(b)
- Then $2 + a^2 = 2 + b^2$
- So $a^2 = b^2$
- Now square root each side
- $\sqrt{a^2} = \sqrt{b^2}$
- So |a| = |b|
- Thus a = $\pm$b or b = $\pm$a
- So now the SAME output value can be produced by different input values ➔ NOT one to one!!

(D) Existence of Inverses

4. Show that $f(x) = \frac{x}{\sqrt{x^2+1}}, x \in \mathbb{R}$ is a one–to–one function, hence find its inverse, f^{-1}.

(D) Existence of Inverses

14. (a) Sketch the graph of $f(x) = x - \frac{1}{x}, x > 0$. Does the inverse function, f^{-1} exist? Give a reason for your answer.

(b) Consider the function $g : S \to \mathbb{R}$ where, $g(x) = x - \frac{1}{x}$. Find the two largest sets S so the the inverse function, g^{-1}, exists. Find both inverses and on separate axes, sketch their graphs.

(D) Exponential Functions

- Graph the exponential function y = e^x + 1 using DESMOS
- Now graph the inverse using DESMOS as well ➔ x = e^y +1
- Explore the data tables of each function & the graphs & notice the connections
- Now graph the function y = ln(x – 1) and compare it to the graph of x – 1= e^y
- Explain the point to the natural log function

(D) Exponential Functions

- Find the inverses of the following functions:

$$(a)\ y = e^{2x}$$
$$(b)\ y = e^{x+2}$$
$$(c)\ y = 2e^{x}$$
$$(d)\ y = e^{x} + 2$$
$$(e)\ y = \ln(x+3) - 1$$
$$(f)\ y = 4\ln(2x)$$

(E) Composing with Inverses

- Let f(x) = 2x – 7.
- Determine the inverse of y = f(x)
- Graph both functions DESMOS
- Draw the line y = x. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$. What do you notice?

(E) Composing with Inverses

- Now let $f(x) = x^2 + 2$, **provided that …….. ???**
- Determine the inverse of y = f(x)
- Graph both functions on DESMOS
- Draw the line y = x. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$. What do you notice?

(E) Composing with Inverses

- How can we use this observation?
- Determine the equation of the inverse of

$$f(x) = \frac{5}{x-3}$$

- Verify that your equation for the inverse IS correct (HINT: Composition)