Lesson 9 – Compositions of Functions

IBHL1 Math - Santowski

The **BIG** Picture

- And we are studying this because ....?
- Functions will be a unifying theme throughout the course ➔ so a solid understanding of **what** functions are and **why** they are used and **how** they are used will be very important!
- Sometimes, complicated looking equations can be easier to understand as being combinations of simpler, parent functions

(A) Function Composition

- So we have a way of creating a new function ➔ we can **compose** two functions which is basically a **substitution of one function into another**.
- we have a notation that communicates this idea ➔ if ***f***(***x***) is one functions and ***g***(***x***) is a second function, then the composition notation is ➔ **f o g (x)**

(B) Composition of Functions – Example

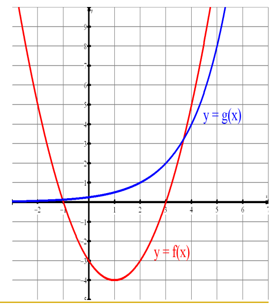
- We will now define *f* and *g* as follows:
- *f* = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}
- We will now work with the composition of these two functions:
  - (i) We will evaluate *fog*(3) (or f(g(3)) and *fog* (1)
  - (ii) evaluate *fog* (5) and see what happens ➔ why?
  - (iii) How does our answer in Q(ii) help explain the idea of "existence"?
  - (iv) evaluate *gof* (9) and *g*(*f*(7)) and *gog* (1)

(B) Composition of Functions – Example

- We can define f and g differently, this time as formulas:
- $f(x) = x^2 - 3$ and $g(x) = 2x + 7$
- We will try the following:
- (i) $f(g(3))$ or $fog(3)$
- (ii) $gof(3)$ or $g(f(3))$
- (ii) $fog(x)$ and $gof(x)$
- (ii) evaluate fog (5)
- (iii) evaluate gof (9) and $g(f(7))$ and gog (1)

(B) Composition of Functions – Example

- We can define f and g differently, this time as graphs:
- We will try the following:
- (i) $f(g(3))$ or $fog(3)$
- (ii) $gof(3)$ or $g(f(3))$
- (iii) evaluate $fog(2)$ and $fog(-1)$
- (iv) evaluate $gof(0)$ and $g(f(1))$ and $gog(2)$



(C) "Existence of Composite"

- Use DESMOS to graph the following functions:

$$f(x) = \sqrt{x-1} \text{ and } g(x) = \ln(x)$$

- State the RANGE of $f(x)$ and of $g(x)$.
- Determine the equation for fog(x)
- Graph the composite function, fog(x) and determine its DOMAIN
- Q(a)? Does fog(x) exist ➔ Why is the answer NO!?!?!?
- Q(b)? Under what domain conditions of g(x) does fog(x) exist

(D) Composition of Functions – Explaining "Existence of Composite"

- Back to our opening example $f(x) = \sqrt{x-1}$ and $g(x) = \ln(x)$

1. Let's consider a mapping diagram. Complete the mapping diagram for the function:

g(x) = ln(x), where the Domain = {0.1, 0.5, 1, 2, e, 5, 8} (ideally {xER|x ≥ 0}

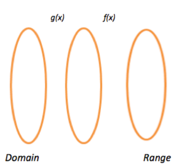
What is the Range of **g(x)** (use calculator) ________________ ? (ideally {yER}

(D) Composition of Functions – Explaining "Existence of Composite"

2. Sometimes functions undergo more than one mapping or transformation ➔ we will do $f \circ g(x)$

Let's consider these two functions: $f(x) = \sqrt{x-1}$ and $g(x) = \ln(x)$, what would this mapping look like below.

Let the Domain = {0.1, 0.5, 1, 2, e, 5, 8} , as before



Determine the Range ______________________________

If our data is mapped via g(x) and then mapped by f(x), the question then remains:

(1) When does this doubling mapping work & when doesn't it?

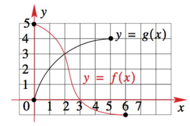
(2) What is the new equation of the twice mapped data that will allow us to get the same result in 1 step?"

Composition, Existance & Domains of Existance

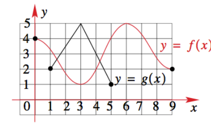
15. Given the functions $f(x) = \sqrt{x^2 - 9}, x \in \mathbf{S}$ and $g(x) = |x| - 3, x \in \mathbf{T}$, find the largest positive subsets of $\mathbb{R}$ so that (a) $g o f$ exists (b) $f o g$ exists.

16. For each of the following functions
(a) determine if $f o g$ exists and sketch the graph of $f o g$ when it exists.
(b) determine if $g o f$ exists and sketch the graph of $g o f$ when it exists.

i.



ii.



Composition, Existance & Domains of Existance

18. The functions f and g are given by $f(x) = \begin{cases} \sqrt{x-1} & \text{if } x \geq 1 \\ x - 1 & \text{if } 0 < x < 1 \end{cases}$ and $g(x) = x^2 + 1$.

(a) Show that $f o g$ is defined. (b) Find $(f o g)(x)$ and determine its range.

19. Let $f : \mathbb{R}^+ \mapsto \mathbb{R}^+$ where $f(x) = \begin{cases} \dfrac{1}{x^2}, & 0 < x \leq 1 \\ \dfrac{1}{\sqrt{x}}, & x > 1 \end{cases}$.

(a) Sketch the graph of f.
(b) Define the composition $f o f$, justifying its existence.
(c) Sketch the graph of $f o f$, giving its range.

Composition, Existance & Domains of Existance

3. All of the following functions are mappings of $\mathbb{R} \mapsto \mathbb{R}$ unless otherwise stated.
(a) Determine the composite functions $(f o g)(x)$ and $(g o f)(x)$, if they exist.
(b) For the composite functions in (a) that do exist, find their range.

i. $f(x) = x + 1, g(x) = x^3$
ii. $f(x) = x^2 + 1, g(x) = \sqrt{x}, x \geq 0$
iii. $f(x) = (x+2)^2, g(x) = x - 2$
iv. $f(x) = \frac{1}{x}, x \neq 0, g(x) = \frac{1}{x}, x \neq 0$,
v. $f(x) = x^2, g(x) = \sqrt{x}, x \geq 0$
vi. $f(x) = x^2 - 1, g(x) = \frac{1}{x}, x \neq 0$
vii. $f(x) = \frac{1}{x}, x \neq 0, g(x) = \frac{1}{x^2}, x \neq 0$
viii. $f(x) = x - 4, g(x) = |x|$
ix. $f(x) = x^3 - 2, g(x) = |x + 2|$
x. $f(x) = \sqrt{4 - x}, x \leq 4, g(x) = x^2$
xi. $f(x) = \frac{x}{x+1}, x \neq -1, g(x) = x^2$
xii. $f(x) = x^2 + x + 1, g(x) = |x|$

(F) Composition of Functions – Example

- For the following pairs of functions
- (a) Determine fog (x)
- (b) Determine gof (x)
- (c) Graph the original two functions in a square view window & make observations about the graph ➔ then relate these observations back to the composition result

$(a)\ f(x) = 3x - 6$ and $g(x) = \frac{1}{3}x + 2$

$(b)\ f(x) = \frac{1}{x+3}$ and $g(x) = \frac{1-3x}{x}$

$(c)\ f(x) = 3 - (x+2)^2$ where $x \geq -2$ and $g(x) = \sqrt{3-x} - 2$

$(d)\ f(x) = e^{2x-1}$ and $g(x) = \frac{1}{2}(\ln(x) - 1)$