Lesson 7 – Function Analysis – Algebraic Perspective

IBL1 Math - Santowski

Fast Five

- Predict the domains & ranges of the following functions. Include a justification for your chosen D & R
- $$f(x) = 1 - \sqrt{5-2x}$$
  $$g(x) = \frac{|x-3|}{1-2x}$$

Lesson Objectives

- Consolidate our understanding of the key features of the parent functions studied in Lesson 6
- Extend our knowledge of these key features by now considering ALGEBRAIC COMBINATIONS of these parent functions
- Finally, consider various algebraic strategies for analyzing the key features of these functions. We will algebraically investigate:
- (i) domain & range
- (ii) symmetries (even & odd)
- (iii) end behavior
- (iv) asymptotic behavior
- (v) intercepts

BIG PICTURE

- Each type of function that we will be studying in this course will have some **features common** with other types of functions BUT will also have some features **unique** to itself
- How can we efficiently use our knowledge of these **key features** to make the algebraic anaysis of a myriad of functions that much easier?

(A) Domain and Range

- Quick recap ➔ which of our parent functions have domain restrictions? Explain why.
- Quick recap ➔ which of our parent functions have range restrictions? Explain why.

(A) Domain and Range (ANS)

- Quick recap ➔ which of our parent functions have domain restrictions? (sqrt(x), 1/x, 1/x^2)
- Quick recap ➔ which of our parent functions have range restrictions? (abs(x), sqrt(x), x2,1/x, 1/x^2)

(A) Domain & Range

- HINT: when looking at the Domain of "complex" functions, rather than asking yourself what the domain IS, ask yourself rather, what the domain ISN'T!!
- For example:

If $f(x)=\sqrt{x-3}$ and $g(x)=\dfrac{1}{2-x}$,

(a) State the domain and range of $y=f(x)$ and $y=g(x)$

(b) State the domain of $y=f\circ g(x)$

(c) State the domain of $y=g\circ f(x)$

(A) Domain & Range

- HINT: when looking at the Domain of "complex" functions, rather than asking yourself what the domain IS, ask yourself rather, what the domain ISN'T!!
- For example, state the domain and range of the following functions:

(a) $f(x)=2-|3-x|$

(b) $g(x)=\dfrac{2x+5}{x+2}$

(c) $h(x)=-3x^2+6x-1$

(d) $k(x)=\dfrac{1}{\sqrt{|x-4|}}$

(A) Domain & Range

7. Determine the domain of each function.

a) $y=3-5x$

b) $y=x^2-4$

c) $y=-2x^2-8x$

d) $y=\sqrt{-x}$

e) $y=\sqrt{5x-\frac{\sqrt{3}}{7}}$

f) $y=\sqrt{x^2-4}$

g) $y=\dfrac{x^2-9}{x-3}$

h) $y=\dfrac{x-3}{x^2-9}$

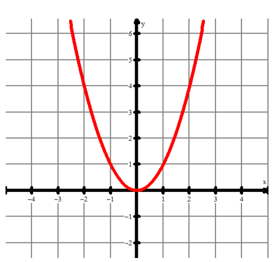
i) $y=\sqrt[4]{\dfrac{x^2+3x+2}{4-x^2}}$

(B) Symmetries

- The two most common symmetries that we will consider for functions will be:
- (a) symmetrical about the y-axis (called EVEN symmetry)
- (b) symmetrical about about the origin (called ODD symmetry – two fold rotational symmetry))
- Q ➔ How do we ALGEBRAICALLY determine this??

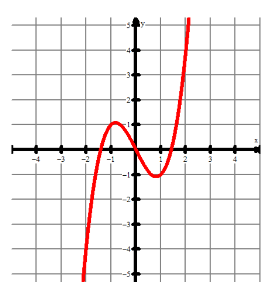
(B) Symmetries

- Consider $y = x^2$
- Notice what happens when you evaluate f(2) and f(-2) ➔
- So EXPLAIN why we make the statement that to test for EVEN symmetry, we state that f(x) = f(-x) for all values of x.



(B) Symmetries

- Consider $y = x^3 - 2x$
- Notice what happens when you evaluate f(2) and f(-2) ➔
- So EXPLAIN why we make the statement that to test for ODD symmetry, we state that f(x) = -f(-x) for all values of x.



(B) Symmetries

- Test the following functions for having either even, odd or neither symmetry:

$$(a)\ f(x) = |x-2|$$
$$(b)\ f(x) = |x| - 2$$
$$(c)\ f(x) = x^2 - x - 4$$
$$(d)\ f(x) = \sqrt{x^2 + 2}$$
$$(e)\ f(x) = \frac{x}{x-2}$$
$$(f)\ f(x) = 2x^3 - 4x$$
$$(g)\ f(x) = -\frac{3}{2x}$$

(B) Symmetries

6. Determine the symmetry of the following. Use proper "proof" format.

a) $f(x) = 4x^3 + \pi x - \sqrt[5]{2x}$

b) $f^{-1}(x) = 3x^{100} - \frac{5}{4} + x^{-2}$

c) $f(x) = \frac{\sqrt{|x|} + \sqrt[3]{x^2}}{x}$

d) $f(x) = \frac{1}{x^2 - x}$

e) $f(x) = \frac{3x}{5 - x^2}$

f) $h(x) = \frac{x^{-1} - x^{\frac{1}{3}}}{x^2 - x^{-2}} - 2$

(C) End Behaviour

- Here we shall simply ask ourselves the question ➔ what happens at the "positive" and "negative" ends of a function (we could be looking for Horizontal Asymptotes here as well)
- So in symbols, as we've described, as x➔+∞ and as x ➔ -∞ (as our domain elements get infinitely larger, both negatively so & positively so
- A couple of key algebraic ideas ➔ what does $(\infty)^{-x}$ "equal" and ∞^x "equal" and (∞ + 2) "equal"

(C) End Behaviour

- Predict the "end behaviours" of the following functions:

$$(a)\ g(x) = x^3 - x^2$$
$$(b)\ g(x) = \frac{2}{x-3}$$
$$(c)\ g(x) = \frac{4x-1}{2x}$$
$$(d)\ g(x) = 4 - 2^{x+1}$$
$$(e)\ g(x) = x^4 - \sqrt{2x}$$
$$(f)\ g(x) = \frac{-2}{x^2}$$

(D) Vertical Asymptotes

- Again, we will narrow down our analysis, because, for now, only two of our parent functions have vertical asymptotes (1/x and 1/x2)
- Question is WHY do they have VAs?
- And then, how can I algebraically predict WHERE the VA's are?

(D) Vertical Asymptotes

- Since we have an idea as to WHY VA's occur, let's predict algebraically where they are in the following functions:

$$(a)\ f(x) = \frac{150}{2x-6}$$
$$(b)\ f(x) = \frac{x}{3x+6}$$
$$(c)\ f(x) = \ln(4+x)$$
$$(d)\ f(x) = \frac{x+1}{\sqrt{3-x}}$$
$$(e)\ f(x) = \frac{2}{x^2-4}$$