Lesson 3 – Introduction to Series

Math HL1 - Santowski

Fast Five

- PART 1
- Find the sum of the first 100 natural numbers
- Outline a way to solve this problem and then carry out your plan
- PART 2
- Determine the sum of the first 17 multiples of 3, starting with term #1 being 1.

(A) Review

- A sequence is a set of ordered terms, possibly related by some pattern
- One such pattern is called arithmetic because each pair of consecutive terms has a common difference
- The general term of an arithmetic sequence is defined by the formula $u_n = u_1 + (n - 1)d$
- A geometric sequence is one in which the consecutive terms differ by a common ratio
- The general term of an geometric sequence is defined by the formula $u_n = u_1 \cdot r^{(n-1)}$

(B) Arithmetic Series

- A **series** is defined as the sum of the terms of a sequence.
- So in this case our sequence was 1,2,3,4,5, …… 99,100
- As an opening exercise, you started by trying to find the sum of the first 100 numbers:
- Symbolically, $S_{100} = 1 + 2 + 3 + 4 + 5 + ….. + 100$

(B) Arithmetic Series - PATTERNS

- A **series** is defined as the sum of the terms of a sequence.
- Here is an easy way to set it up:

S_{100}	1	2	3	4	5	…….	99	100
S_{100}	100	99	98	97	96	……	2	1
$2S_{100}$	101	101	101	101	101	……	101	101

- So then the sum is (101)(100)÷2

(B) Arithmetic Series

- For an arithmetic sequence then the formula for the sum of its terms is:

$$Sn = \frac{n}{2}\left(u_1 + u_n\right) = \frac{n}{2}\left(2u_1 + \left(n-1\right)d\right)$$

(C) Examples

- Ex 1. Find the sum of the series 13 + 24 + 35 + … + 156
- Ex 2. For the series 2 + 11 + 20 + 29 + ..., find u_{20} and S_{20}
- Ex 3. The fifth term of an arithmetic series is 9 and the sum of the first 16 is 480. Find the first three terms of the series.
- Ex 4. In an arithmetic series of 50 terms, the 17th term is 53 and the 28th term is 86. Find the sum of the series.

(C) Examples

- ex 5. Shayla deposited \$128 into her account. Each week she deposits \$7 less than the previous week until she deposits her last deposit of \$2. What total amount did she deposit?
- ex 6. Jayne buys 10 widgets on the Jan 1st , 15 on the 1st of Feb, 20 on the 1st of March, etc..... How many widgets has she acquired in 2 years? How long does it take her to acquire 5,000 widgets?

Lesson Part 2 - Fast Five

- Determine the sum of the first 17 multiples of 3, starting with term #1 being 1
- Symbollically, you are trying to determine:
- S_{17} = 1 + 3 + 9 + 27 + 81 + 243 + ...
- NO CALCULATOR

(D) Geometric Series - PATTERNS

- Find the sum of the first 7 terms of the series
- S_7 = 1 + 3 + 9 + 27 + 81 + 243 + 729
- $S_7 = a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6$

S_7	1	3	3^2	3^3	3^4	3^5	3^6	
$3S_7$		3	3^2	3^3	3^4	3^5	3^6	3^7

- $3S_7 - S_7 = 2S_7 = (3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7) - (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6) = 3^7 - 1$
- $S_7 = ½(3^7 - 1)$

(D) Geometric Series

- So in general, the formula for the sum of a geometric series is:

$$S_n = \frac{(u_{n+1} - u_1)}{r-1} = \frac{u_1(r^n - 1)}{r-1}$$

(E) Examples

- ex 1. Find S_8 given:
- (a) 2 - 6 + 18 - 54 + ...
- (b) 200 + 100 + 50 + 25 + ...
- ex 2. Find the total amount you make if you were paid a rupee a day, but the amount was doubled every day for a month
- ex 3. Find the sum 1/16 + 1/4 + 1 + 4 + ... + 65536

(E) Examples

- Ex 4. The fifth term of a geometric series is 405 and the sixth term is 1215. Find the sum of the first nine terms.
- ex 5. A ball drops from a height of 16 m and its height on the bounce is 5/8th of the previous maximum height. Determine the total height bounced by the ball after it touches the ground for the 7th bounce.

(F) Examples – Partial Sums & Infinite Series

- Given the series S = 200 + 100 + 50 + 25 …, which is an example of an infinite geometric series
- (i) Determine the first 4 partial sums of the series (determine each of S_1, S_2, S_3, S_4)
- (ii) Determine the seven partial sum (S_7)
- (iii) Determine S_{13}
- (iv) Predict $S_{1,000,000}$

(F) Examples – Sums Formula & Infinite Series

- Given the series S = 200 + 100 + 50 + 25 …, which is an example of an infinite geometric series
- (i) Use the sum formula to determine S_4
- (ii) Use the sum formula to determine S_7
- (iii) Use the sum formula to determine S_{13}
- (iv) Use the sum formula to predict $S_{1,000,000}$

(F) Examples – Infinite Series

- The series ½ + ¼ + 1/8 + 1/16 + ……. is an example of an infinite geometric series.
- (a) Determine the sum of this series.
- (b) Is it possible to find the sum of *any infinite geometric sequence? Explain.*
- (c) Under what conditions is it possible to find the sum of an infinite geometric sequence

(F) Examples – Infinite Series

- Show that the sum of *n terms of the series 2 + 1 + ½ + ¼ + ….. u_n is always* less than 4, where *n is any natural number.*

(I) Internet Links

- Geometric Sequences & Series From West Texas A&M
- Arithmetic Sequences & Series From West Texas A&M U