Oxford text Ex 1E Q9 9/3/14

Q9 infinite series $S = \frac{u_1}{1-r}$

$S - S_n = K u_n$ $\quad S_n = \frac{u_1(1-r^n)}{1-r}$

(i) Find r: $S - S_n = K u_n$

$$\frac{u_1}{1-r} - \frac{u_1(1-r^n)}{1-r} = K u_n$$

$$\frac{\cancel{u_1} - \cancel{u_1} + u_1 r^n}{1-r} = K u_1 r^{n-1}$$

$\times(1-r)$ $\quad \frac{u_1 r^n}{1-r} = K u_1 r^{n-1}$

$$u_1 r^n = K u_1 r^{n-1}(1-r)$$

$r^{n-1} \cdot r = r^n$ $\quad u_1 r^n = K u_1 r^{n-1} - K u_1 r^n$

$+ K u_1 r^n$ $\quad u_1 r^n + K u_1 r^n = K u_1 r^{n-1}$

$\div u_1$ $\quad r^n + K r^n = K r^{n-1}$

$$\frac{r^n(1+K)}{1+K} = \frac{K \cdot r^{n-1}}{1+K}$$

$\div r^{n-1}$ $\quad r^n = \left(\frac{K}{1+K}\right) r^{n-1}$

$$\frac{r^n}{r^{n-1}} = \frac{K}{1+K}$$

$$r^{n-(n-1)} = \frac{K}{1+K}$$

$$\boxed{r = \frac{K}{1+K}}$$

Hence → means use $r = {}^{K}/_{K+1}$ in your work

(ii) $$S = \frac{u_1\left(1 - \left(\frac{K}{K+1}\right)^n\right)}{1 - \left(\frac{K}{K+1}\right)} + Ku_1\left(\frac{K}{K+1}\right)^{n-1}$$

$$S = u_1 \frac{\left(\frac{(K+1)^n}{(K+1)^n} - \frac{K^n}{(K+1)^n}\right)}{\left(\frac{K+1}{K+1}\right) - \frac{K}{K+1}} + Ku_1\left(\frac{K^{n-1}}{(K+1)^{n-1}}\right)$$

$$S = \frac{u_1\left(\frac{(K+1)^n - K^n}{\cancel{(K+1)^n}}\right)}{\cancel{\frac{1}{K+1}}} + \frac{u_1\left(K^n\right)}{(K+1)^{n-1}}$$

$$S = u_1\left(\frac{(K+1)^n - K^n}{(K+1)^{n-1}}\right) + u_1\left(\frac{K^n}{(K+1)^{n-1}}\right)$$

$$S = u_1\left(\frac{(K+1)^n \cancel{- K^n} \cancel{+ K^n}}{(K+1)^{n-1}}\right)$$

$$S = u_1\left(\frac{(K+1)^n}{(K+1)^{n-1}}\right)$$

$$S = u_1\left((K+1)^{n-(n-1)}\right)$$

$$S = u_1(K+1)$$