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term n  $k+4$   
 term n+1  $5k+4$   
 term n+2  $k+20$

$$\text{so } \frac{5k+4}{k+4} = \text{ratio} = \frac{k+20}{5k+4}$$

$$\text{so } \frac{5k+4}{k+4} = \frac{k+20}{5k+4}$$

$$(5k+4)^2 = (k+4)(k+20)$$

$$25k^2 + 40k + 16 = k^2 + 24k + 80$$

$$24k^2 + 16k - 64 = 0$$

$$8(3k^2 + 2k - 8) = 0$$

$$8(3k-4)(k+2) = 0$$

$$\therefore k = \frac{4}{3} \text{ or } k = -2$$

Q7  $u_1 + u_3 = 40$   $u_6 = ?$   
 $u_2 + u_4 = 96$

$$u_3 = u_1 \times r \times r = u_1 r^2 \Rightarrow u_1 + u_1 r^2 = 40$$

$$u_4 = u_2 \times r \times r = u_2 r^2 \Rightarrow u_2 + u_2 r^2 = 96$$

$$\text{so } \begin{cases} u_1(1+r^2) = 40 \\ u_2(1+r^2) = 96 \end{cases}$$

$$\therefore \frac{u_2(1+r^2)}{u_1(1+r^2)} = \frac{96}{40}$$

$$\frac{u_2}{u_1} = r = \frac{96}{40} = \frac{12}{5}$$

$$\text{then } \frac{40}{1+r^2} = u_1 \Rightarrow u_1 = \frac{40}{1+(\frac{12}{5})^2} = \frac{1000}{169}$$

$$\text{then } u_6 = u_1 r^5 = \left( \frac{4000}{169} \right) \left( \frac{12}{5} \right)^5 = \frac{248,932,000}{17,528,125}$$

$$8 \quad \begin{aligned} u_1 + u_2 + u_3 &= 35/2 & \Rightarrow u_1 + u_1 r + u_1 r^2 &= 35/2 \\ u_1 \cdot u_2 \cdot u_3 &= 125 & \Rightarrow u_1 \cdot u_1 r \cdot u_1 r^2 &= 125 \\ & & 3u_1^3 r^3 &= 125 \\ & & (u_1 r)^3 &= 125 \\ & & \therefore u_1 r &= 5 \\ & & \{ u_1 &= \frac{5}{r} \end{aligned}$$

$$\text{use } u_1 r = 5 \Rightarrow \text{thys } \begin{aligned} u_1 + 5 + u_1 r^2 &= 35/2 \\ u_1 + u_1 r^2 &= 25/2 \\ u_1 (1+r^2) &= 25/2 \end{aligned}$$

$$\text{Likewise } \begin{aligned} u_1 \times 5 \times u_1 r^2 &= 125 \\ u_1^2 r^2 &= 25 \\ \text{so } u_1 &= \sqrt{\frac{25}{r^2}} = \frac{5}{r} \end{aligned}$$

$$\text{so } \frac{5}{r} + 5 + 5r = \frac{35}{2}$$

$$\begin{aligned} 4 \quad 5 \left( \frac{1}{r} + 1 + r \right) &= \frac{35}{2} \\ \frac{1}{r} + 1 + r &= \frac{7}{2} \end{aligned}$$

$$\left( \frac{1}{r} + r = \frac{5}{2} \right) \cdot 2r$$

$$2 + 2r^2 = 5r$$

$$2r^2 - 5r + 2 = 0$$

$$(2r - 1)(r - 2) = 0$$

$$r = \frac{1}{2} \quad r = 2$$

$$\therefore \begin{aligned} u_1 &= \frac{5}{\frac{1}{2}} \quad \text{or} \quad \frac{5}{2} & \Rightarrow & 10 \\ u_1 &= \frac{5}{2} & \Rightarrow & 5 \end{aligned}$$

HL Math (Unit 8.1.1) A/Seg

10  $u_n = 4 - \sqrt{3}$

$u_{n+1} = x = u_n + d$

$u_{n+2} = y = u_n + 2d = u_{n+1} + d$

$u_{n+3} = 2\sqrt{3} - 2 = u_n + 3d$

so  $3d =$  using first term & 4th term

$3d = (2\sqrt{3} - 2) - (4 - \sqrt{3})$

$3d = 3\sqrt{3} - 6$

$d = \sqrt{3} - 2$

then  $u_{n+1} = x = (4 - \sqrt{3}) + (\sqrt{3} - 2)$

$x = 2$

$u_{n+2}$

$u_{n+2} = y = u_n + 2d = (4 - \sqrt{3}) + 2(\sqrt{3} - 2)$

$y = -\sqrt{3}$

12

$u_3 = x + y$

$u_5 = x - y$

but  $u_5 = u_3 + 2d$

so  $d = \frac{u_5 - u_3}{2}$

$= \frac{(x - y) - (x + y)}{2}$

$d = -y$

then

$u_{12} = u_5 + 7d$

$= (x - y) + 7(-y)$

$u_{12} = x - 8y$

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$$u_5 + 2u_3 = u_{12}$$

$$u_7 = 25$$

$$u_n = ?$$

$$u_5 = u_1 + 4d$$

$$u_3 = u_1 + 2d$$

$$u_{12} = u_1 + 11d$$

$$u_7 = u_1 + 6d$$

$$\text{So } (u_1 + 4d) + 2(u_1 + 2d) = u_1 + 11d$$

$$2u_1 = 3d$$

$$\therefore u_1 = \frac{3}{2}d$$

$$\text{then } u_7 = u_1 + 6d = 25$$

$$\frac{3}{2}d + 6d = 25$$

$$3d + 12d = 50$$

$$d = 50/15 = \frac{10}{3}$$

$$\text{So } u_1 = \frac{3}{2} \times \frac{10}{3} = 5$$

$$\therefore u_n = 5 + (n-1) \frac{10}{3}$$

$$u_n = \frac{10}{3}n - \frac{5}{3}$$