

Normal Distribution

Exercises (calculator allowed on all questions except 4 & 6)

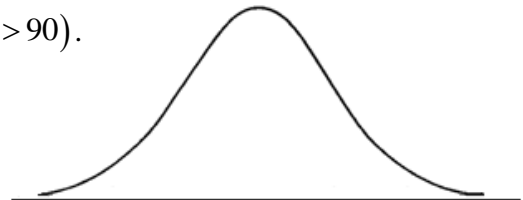
[answers included]

- The age of trainees in a company's training program is normally distributed with mean $\mu = 27.3$ and standard deviation $\sigma = 3.1$. If a student is chosen at random:
 - what is the probability that a trainee is younger than 30 years old?
 - what percentage of trainees are between the ages of 24 and 28?
 - what percentage of trainees are within one standard deviation of the mean?
- The mean rainfall in Chicago for the month of May is 9.52 cm with a standard deviation of 1.84 cm. Over a 50 year period, assuming the rainfall distribution is normal, how many times for Chicago would you expect there to be:
 - more than 10 cm of rain in May?
 - between 9 and 10 cm of rain in May?
- A random variable X is distributed normally with a mean of 15 and a standard deviation of 3.
 - Find $P(X \leq 12.5)$.
 - Find $P(X \geq 20.5)$.
 - Let $P(X \leq a) = 0.65$. Find the value of a .
 - Let $P(b \leq X \leq 20.5) = 0.5$. Find the value of b .

[**No** GDC]

- Let X be a random variable that is normally distributed with a mean of 100 seconds and a standard deviation of 10 seconds.

(a) On the diagram, shade the region representing $P(X > 90)$.



- (b) Given that $P(X < k) = P(X > 90)$, find the value of k .
- (c) Given that $P(X > 90) = 0.84$ (correct to two significant figures), find $P(90 < X < k)$.

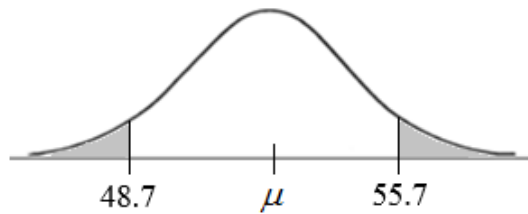
- X is a random variable that is normally distributed with a mean of 5.
 - Given that $P(X < 4) = 0.1057$, show that the value of the standard deviation (accurate to three significant figures) is 0.8.
 - Find $P(|X - 5| < 0.5)$

[**No** GDC]

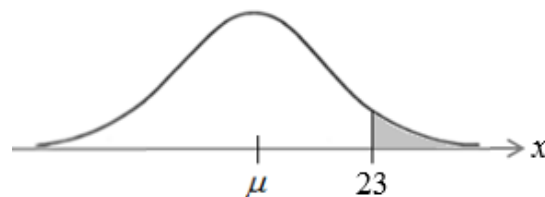
- A random variable X is distributed normally with a mean of 30 and a variance of 9.
 - Find the value of x that is 1.4 standard deviations **above** the mean.
 - Find the value of x that is 1.4 standard deviations **below** the mean.

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7. The diagram below shows the probability density function for a random variable X that is distributed normally. Each of the shaded regions has an area of 0.08.



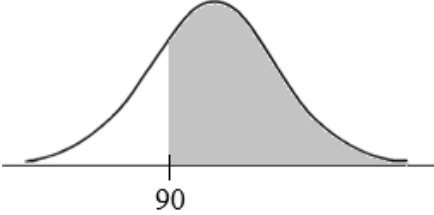
- (a) Find the value of the mean, μ , of X .
- (b) Find the value of the standard deviation, σ , of X .
8. The heights of a certain type of cactus plant follow a normal distribution. It is known that 30% of these plants have a height less than 32 cm and 15% have a height greater than 44 cm. Find the value of the mean μ and the value of the standard deviation σ .
9. On the label of a bottle of lemonade it states that the volume of lemonade is 500 ml. For all the bottles of lemonade produced by the manufacturer the volume of lemonade is normally distributed with mean 500 ml and standard deviation of 2.3 ml.
- (a) Find the probability that the volume of lemonade in a randomly chosen bottle is less than 498 ml.
- (b) If the percentage of bottles containing more than k ml of lemonade is found to be 5%, find the value of k to the nearest tenth of a ml.
10. The diagram below shows a normal curve for the random variable X with mean μ and standard deviation σ . The shaded region has an area of 0.18.



- (a) Given that $P(X \leq 15) = 0.18$, write down the value of the mean μ .
- (b) Find the value of the standard deviation σ .
11. The masses of individual apples sold in a food store are normally distributed. The supplier who provides the store with apples knows that 75% of the apples have a mass greater than 85 grams and that 10% of the apples have a mass greater than 120 grams.
- (a) Find the value of the mean μ and the value of the standard deviation σ .
- The apples are always sold in bags containing 6 apples.
- (b) Find the probability that each apple in a randomly selected bag has a mass less than 105 grams.
- (c) How many apples (to the nearest whole number) from a randomly selected bag would you expect to have a mass greater than 90 grams?

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ANSWERS

1. (a) 0.808 (b) 44.6% (c) 68.3%
2. (a) 20 times (b) 10 times
3. (a) 0.202 (b) 0.0334 (c) $a \approx 16.2$ (d) $b \approx 14.7$
4. (a)


(b) $k = 110$ (c) $P(90 < X < k) = 0.68$
5. (b) 0.468
6. (a) $x = 34.2$ (b) $x = 25.8$
7. (a) $\mu = 52.2$ (b) $\sigma \approx 2.49$
8. $\mu \approx 36.0$ cm, $\sigma \approx 7.69$ cm
9. (a) 19.2% (b) 503.8 ml
10. (a) $\mu = 19$ (b) $\sigma \approx 4.37$
11. (a) $\mu \approx 97.1$ grams, $\sigma \approx 17.9$ grams
(b) 0.0914
(c) 4 apples