International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Diploma Programme

Mathematics HL guide

First examinations 2014

Topic 9—Option: Calculus

48 hours

The aims of this option are to introduce limit theorems and convergence of series, and to use calculus results to solve differential equations.

	Content	Further guidance	Links
9.1	Infinite sequences of real numbers and their convergence or divergence.	Informal treatment of limit of sum, difference, product, quotient; squeeze theorem. Divergent is taken to mean not convergent.	**TOK:** Zeno's paradox, impact of infinite sequences and limits on our understanding of the physical world.
9.2	Convergence of infinite series. Tests for convergence: comparison test; limit comparison test; ratio test; integral test.	The sum of a series is the limit of the sequence of its partial sums. Students should be aware that if $\lim_{x\to\infty} x_n = 0$ then the series is not necessarily convergent, but if $\lim_{x\to\infty} x_n \neq 0$, the series diverges.	**TOK:** Euler's idea that $1-1+1-1+\ldots=\frac{1}{2}$. Was it a mistake or just an alternative view?
	The p-series, $\sum \frac{1}{n^p}$.	$\sum \frac{1}{n^p}$ is convergent for $p>1$ and divergent otherwise. When $p=1$, this is the harmonic series.	
	Series that converge absolutely. Series that converge conditionally.	Conditions for convergence.	
	Alternating series. Power series: radius of convergence and interval of convergence. Determination of the radius of convergence by the ratio test.	The absolute value of the truncation error is less than the next term in the series.	

	Content	Further guidance	Links
9.3	Continuity and differentiability of a function at a point. Continuous functions and differentiable functions.	Test for continuity: $\lim_{x\to a-} f(x) = f(a) = \lim_{x\to a+} f(x)$. Test for differentiability: f is continuous at a and $\lim_{h\to 0-} \frac{f(a+h)-f(a)}{h}$ and $\lim_{h\to 0+} \frac{f(a+h)-f(a)}{h}$ exist and are equal. Students should be aware that a function may be continuous but not differentiable at a point, eg $f(x)=\|x\|$ and simple piecewise functions.	
9.4	The integral as a limit of a sum; lower and upper Riemann sums. Fundamental theorem of calculus. Improper integrals of the type $\int_a^{\infty} f(x)\,dx$.	 $\frac{d}{dx}\left[\int_a^x f(y)\,dy\right] = f(x)$.	**Int:** How close was Archimedes to integral calculus? **Int:** Contribution of Arab, Chinese and Indian mathematicians to the development of calculus. **Aim 8:** Leibniz versus Newton versus the “giants” on whose shoulders they stood—who deserves credit for mathematical progress? **TOK:** Consider $f(x)=\frac{1}{x}$, $1 \le x \le \infty$. An infinite area sweeps out a finite volume. Can this be reconciled with our intuition? What does this tell us about mathematical knowledge?

	Content	Further guidance	Links
9.5	First-order differential equations. Geometric interpretation using slope fields, including identification of isoclines. Numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$ using Euler's method. Variables separable. Homogeneous differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = f\left(\frac{y}{x}\right)$ using the substitution $y = vx$. Solution of $y' + P(x)y = Q(x)$, using the integrating factor.	$y_{n+1} = y_n + hf(x_n, y_n)$, $x_{n+1} = x_n + h$, where h is a constant.	**Appl:** Real-life differential equations, eg Newton's law of cooling, population growth, carbon dating.
9.6	Rolle's theorem. Mean value theorem. Taylor polynomials; the Lagrange form of the error term. Maclaurin series for e^x, $\sin x$, $\cos x$, $\ln(1+x)$, $(1+x)^p$, $p \in \mathbb{Q}$. Use of substitution, products, integration and differentiation to obtain other series. Taylor series developed from differential equations.	Applications to the approximation of functions; formula for the error term, in terms of the value of the $(n+1)^{\text{th}}$ derivative at an intermediate point. Students should be aware of the intervals of convergence.	**Int, TOK:** Influence of Bourbaki on understanding and teaching of mathematics. **Int:** Compare with work of the Kerala school.

	Content	Further guidance	Links
9.7	The evaluation of limits of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ and $\lim_{x \to \infty} \frac{f(x)}{g(x)}$. Using l'Hôpital's rule or the Taylor series.	The indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$. Repeated use of l'Hôpital's rule.	