International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Diploma Programme

Mathematics HL guide

First examinations 2014

Topic 3—Core: Circular functions and trigonometry 22 hours

The aims of this topic are to explore the circular functions, to introduce some important trigonometric identities and to solve triangles using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated, for example, by $x \mapsto \sin x^{\circ}$.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles. Length of an arc; area of a sector.	Radian measure may be expressed as multiples of π, or decimals. Link with 6.2.	**Int:** The origin of degrees in the mathematics of Mesopotamia and why we use minutes and seconds for time. **TOK:** Mathematics and the knower. Why do we use radians? (The arbitrary nature of degree measure versus radians as real numbers and the implications of using these two measures on the shape of sinusoidal graphs.) **TOK:** Mathematics and knowledge claims. If trigonometry is based on right triangles, how can we sensibly consider trigonometric ratios of angles greater than a right angle? **Int:** The origin of the word "sine". **Appl:** Physics SL/HL 2.2 (forces and dynamics). **Appl:** Triangulation used in the Global Positioning System (GPS). **Int:** Why did Pythagoras link the study of music and mathematics? **Appl:** Concepts in electrical engineering. Generation of sinusoidal voltage.
3.2	Definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ in terms of the unit circle. Exact values of sin, cos and tan of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. Definition of the reciprocal trigonometric ratios $\sec\theta$, $\csc\theta$ and $\cot\theta$. Pythagorean identities: $\cos^2\theta + \sin^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \csc^2\theta$.		
3.3	Compound angle identities. Double angle identities. **Not required:** Proof of compound angle identities.	Derivation of double angle identities from compound angle identities. Finding possible values of trigonometric ratios without finding θ, for example, finding $\sin 2\theta$ given $\sin\theta$.	

(continued)

	Content	Further guidance	Links
3.4	Composite functions of the form $f(x) = a\sin(b(x+c)) + d$. Applications.		*(see notes above)* **TOK:** Mathematics and the world. Music can be expressed using mathematics. Does this mean that music is mathematical, that mathematics is musical or that both are reflections of a common "truth"? **Appl:** Physics SL/HL 4.1 (kinematics of simple harmonic motion).
3.5	The inverse functions $x \mapsto \arcsin x$, $x \mapsto \arccos x$, $x \mapsto \arctan x$; their domains and ranges; their graphs.		
3.6	Algebraic and graphical methods of solving trigonometric equations in a finite interval, including the use of trigonometric identities and factorization. **Not required:** The general solution of trigonometric equations.		**TOK:** Mathematics and knowledge claims. How can there be an infinite number of discrete solutions to an equation?
3.7	The cosine rule The sine rule including the ambiguous case. Area of a triangle as $\frac{1}{2}ab\sin C$.		**TOK:** Nature of mathematics. If the angles of a triangle can add up to less than 180°, 180° or more than 180°, what does this tell us about the "fact" of the angle sum of a triangle and about the nature of mathematical knowledge?
	Applications.	Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	**Appl:** Physics SL/HL 1.3 (vectors and scalars); Physics SL/HL 2.2 (forces and dynamics). **Int:** The use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.