International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Diploma Programme

Mathematics HL guide

First examinations 2014

Topic 2—Core: Functions and equations

22 hours

The aims of this topic are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic.

	Content	Further guidance	Links
2.1	Concept of function $f : x \mapsto f(x)$: domain, range; image (value). Odd and even functions.		**Int:** The notation for functions was developed by a number of different mathematicians in the 17th and 18th centuries. How did the notation we use today become internationally accepted?
	Composite functions $f \circ g$.	$(f \circ g)(x) = f(g(x))$. Link with 6.2.	**TOK:** The nature of mathematics. Is mathematics simply the manipulation of symbols under a set of formal rules?
	Identity function.		
	One-to-one and many-to-one functions.	Link with 3.4.	
	Inverse function f^{-1}, including domain restriction. Self-inverse functions.	Link with 6.2.	

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2.2	The graph of a function; its equation $y = f(x)$. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes and symmetry, and consideration of domain and range. The graphs of the functions $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$. The graph of $y = \dfrac{1}{f(x)}$ given the graph of $y = f(x)$.	 Use of technology to graph a variety of functions.	**TOK:** Mathematics and knowledge claims. Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically (analytically)? **Appl:** Sketching and interpreting graphs; Geography SL/HL (geographic skills); Chemistry 11.3.1. **Int:** Bourbaki group analytical approach versus Mandlebrot visual approach.
2.3	Transformations of graphs: translations; stretches; reflections in the axes. The graph of the inverse function as a reflection in $y = x$.	Link to 3.4. Students are expected to be aware of the effect of transformations on both the algebraic expression and the graph of a function.	**Appl:** Economics SL/HL 1.1 (shift in demand and supply curves).
2.4	The rational function $x \mapsto \dfrac{ax+b}{cx+d}$, and its graph. The function $x \mapsto a^x$, $a > 0$, and its graph. The function $x \mapsto \log_a x$, $x > 0$, and its graph.	The reciprocal function is a particular case. Graphs should include both asymptotes and any intercepts with axes. Exponential and logarithmic functions as inverses of each other. Link to 6.2 and the significance of e. Application of concepts in 2.1, 2.2 and 2.3.	 **Appl:** Geography SL/HL (geographic skills); Physics SL/HL 7.2 (radioactive decay); Chemistry SL/HL 16.3 (activation energy); Economics SL/HL 3.2 (exchange rates).

	Content	Further guidance	Links
2.5	Polynomial functions and their graphs. The factor and remainder theorems. The fundamental theorem of algebra.	The graphical significance of repeated factors. The relationship between the degree of a polynomial function and the possible numbers of x-intercepts.	
2.6	Solving quadratic equations using the quadratic formula. Use of the discriminant $\Delta = b^2 - 4ac$ to determine the nature of the roots. Solving polynomial equations both graphically and algebraically. Sum and product of the roots of polynomial equations. Solution of $a^x = b$ using logarithms. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.	May be referred to as roots of equations or zeros of functions. Link the solution of polynomial equations to conjugate roots in 1.8. For the polynomial equation $\sum_{r=0}^{n} a_r x^r = 0$, the sum is $\frac{-a_{n-1}}{a_n}$, the product is $\frac{(-1)^n a_0}{a_n}$.	**Appl:** Chemistry 17.2 (equilibrium law). **Appl:** Physics 2.1 (kinematics). **Appl:** Physics 4.2 (energy changes in simple harmonic motion). **Appl:** Physics (HL only) 9.1 (projectile motion). **Aim 8:** The phrase “exponential growth” is used popularly to describe a number of phenomena. Is this a misleading use of a mathematical term?

	Content	Further guidance	Links
2.7	Solutions of $g(x) \geq f(x)$. Graphical or algebraic methods, for simple polynomials up to degree 3. Use of technology for these and other functions.		