Precalculus: Trig Assignment #6 /100 Name: __________________

Answer the following questions without a calculator on graph paper/looseleaf.

/8 1. Prove that $\sin(x-\pi) = -\sin x$ using
   a) an algebraic approach b) a graphical approach

/8 2. Prove that a horizontal translation of π to the left of $y=\cos x$ is equivalent to a vertical reflection of $y=\cos x$ using
   a) an algebraic approach b) a graphical approach

/8 3. Prove that $y=\csc x$ and $y=\sec x$ are cofunctions using a graphical approach.

/8 4. Prove that the solutions to $\cos^2 x - \sin^2 x = 0$ for $0 \le x < 2\pi$ are equivalent to the zeroes of $y=\cos 2x$ by solving the equation $\cos^2 x - \sin^2 x = 0$ algebraically and by graphing $y=\cos 2x$. State an identity as a conclusion to your findings for $x \in R$.

/8 5. Prove that the solutions to $\cos^2 x - \sin^2 x = 0$ for $x \in R$ are equivalent to the points of intersection of the horizontal lines $y=\pm 1$ and the trigonometric graph $y=\tan x$ by solving the equation $\cos^2 x - \sin^2 x = 0$ algebraically and by determining the points of intersection graphically.

/8 6. Prove that the solutions to $\sin x \cos x = 0$ for $x \in R$ are equivalent to the zeroes of $y=\frac{1}{2}\sin(2x)$ by solving the equation algebraically and by graphing $y=\frac{1}{2}\sin(2x)$.

/12 7. Graph the function $y = 3\sin x\cos^2 x - \sin^3 x$ for $0 \le x \le 2\pi$; use *xscl* = $\frac{\pi}{6}$.
   a) What is the period of this function? [Hint: How many cycles are there in 2π rad?]
   b) Determine a simpler equation for this function in terms of $\sin x$ and state an identity for $3\sin x\cos^2 x - \sin^3 x$.
   c) Prove the resulting identity algebraically.

8. The following question pertains to variations of a sinusoid curve.

a) Sketch $y = x + \sin x$ and $y = x$ in the following window $[-2\pi \le x \le 2\pi, -2\pi \le y \le 2\pi$, *xscl* = *yscl* = $\frac{\pi}{2}$, *xres* = 2] and answer the following:

(i) Use the TI-89 Intersection Function (F5-5 from the graphing screen) to determine the coordinates of the points of intersection of the two curves on $-2\pi < x < 2\pi$.

(ii) Use an algebraic method to determine *all* intersection points for the two curves (for $x \in R$).

/16

b) Sketch $y = x\sin x$ for each of the following two windows

- $[-2\pi \le x \le 2\pi, -2\pi \le y \le 2\pi$, *xscl* = *yscl* = $\frac{\pi}{2}$, *xres* = 2]
- $[-8\pi \le x \le 8\pi, -8\pi \le y \le 8\pi$, *xscl* = *yscl* = $\frac{\pi}{2}$, *xres* = 2]
- and answer the following questions:

(i) Is $y = x\sin x$ a bounded function? [Hint: Determine a finite number *A* such that $|\sin x| \le A$; then determine a bound (i.e., finite value) for $|x \cdot \sin x| = |x| \cdot |\sin x|$, $x \in R$ if possible.]

(ii) Determine the equations of the slant asymptotes for $y = x\sin x$. Verify these equations by plotting the asymptotes on the second window graph for $y = x\sin x$.

c) Sketch $y = \frac{\sin x}{x}$ in the following window $[-7\pi \le x \le 7\pi, -1 \le y \le 1$, *xscl* = $\frac{\pi}{2}$, *yscl* = 0.25, *xres* = 2] and answer the following questions:

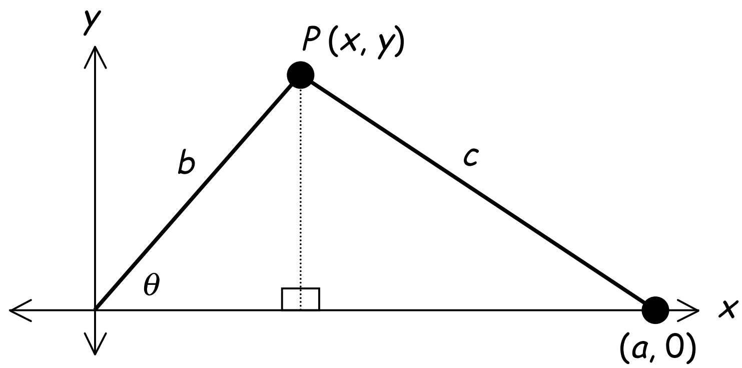
(i) Hypothesize the symmetry of the graph.

(ii) Prove your hypothesis algebraically.

(iii) Regraph $y = \frac{\sin x}{x}$ on the calculator for the new domains $-100 \le x \le 100$ and $-1000 \le x \le 1000$. Do NOT sketch these graphs in your assignment. Use these graphs to determine the type(s) and equation(s) of any asymptotes of $y = \frac{\sin x}{x}$.

9. Prove the Law of Cosines: If any triangle has sides with lengths a, b and c and θ is the angle between the sides of lengths a and b, then $c^2 = a^2 + b^2 - 2ab\cos\theta$.

/8



10. Use the figure below to prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

[Hint 1: Determine c^2 by two different methods (using the cosine law and using the distance formula between A and B) and comparing the two expressions.]
[Hint 2: Determine the coordinates of A and B in terms of α and β in order to calculate c^2 by the distance formula. The points lie on the unit circle.]

/16