

NO CALCULATOR - USE IDENTITIES and leave answers in EXACT FORM.

1. Evaluate the following exactly. State/show all identities required.

a) $\cos(-10\alpha)$, given that $\csc(10\alpha) = \frac{25}{24}$

b) $\sec^2\left(\frac{\beta}{4}\right) - \tan^2\left(\frac{\beta}{4}\right)$, for all $\beta \in \mathbb{R}$

c) $\csc^2(200\beta) - \cot^2(200\beta)$, for all $\beta \in \mathbb{R}$

d) $\sin\left(\frac{\pi}{2} - \theta\right)$, given that $\cot\left(\frac{\pi}{2} - \theta\right) = \frac{5}{12}$

e) $\sin\left(\frac{\pi}{2} - \theta\right)$, given that $\csc\theta = 4$

f) $\sin\left(-\tan^{-1}\frac{1}{3}\right)$

g) $\cos^2\left(\sec^{-1}\frac{4}{3}\right) - \cot\left(\frac{\pi}{2} - \tan^{-1}\frac{1}{3}\right) + \sin^2\left(\sec^{-1}\frac{4}{3}\right)$

2. Given that $\tan 8\alpha = -\frac{2}{5}$ and $\frac{\pi}{2} < 8\alpha < \pi$, evaluate the following exactly:

a) $\cot 8\alpha$

c) $\sin 4\alpha$

e) $\tan 4\alpha$

g) $\sec 16\alpha$

b) $\sec 8\alpha$

d) $\csc 4\alpha$

f) $\cot(-4\alpha)$

h) $\tan(-16\alpha)$

3. Evaluate the following exactly using sum/difference identities and special triangles:

a) $\cos 75^\circ$

c) $\sin 75^\circ$

e) $\tan(\alpha + \beta)$ if $\tan \alpha = \frac{2}{3}$ and $\tan \beta = 2$

b) $\sec(-15^\circ)$

d) $\csc(-15^\circ)$

f) $\cot(\beta - \alpha)$ if $\cos \alpha = \frac{1}{3}$ and $\csc \beta = \frac{5}{3}$

4. Determine that the value of $\cos 195^\circ - \cos 105^\circ$ is $-\frac{\sqrt{2}}{2}$ using:

a) a sum or difference identity and special triangles.

b) a double-angle identity, a half-angle identity and special triangles.

c) a sum-to-product identity and special triangles.

5. Prove the following using the sum and difference identities:

a) $\sin 2\theta = 2\sin\theta \cos\theta$

b) $\cos 2\theta = 2\cos^2\theta - 1$

6. Prove that

a) $\tan\theta = \frac{1 - \cos 2\theta}{\sin 2\theta}$

b) $\frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$