Precalculus: Trig Assignment #2 (Solving Equations)

Answer the following questions on looseleaf/graph paper. No calculator.

PART A – Solving trig equations. For each question, state

(i) the reference angle, if possible
(ii) the quadrants in which the angle lies, if possible
(iii) the angle in radians for one revolution: $0 \le \theta < 2\pi$
(iv) all coterminal angles, using an expression involving $n \in I$: $\theta \in R$

1. Solve the following one-step trig equations:

a) $\cos\theta = -\frac{\sqrt{2}}{2}$ c) $\cot\theta = \text{und}$ e) $\sin\theta = \frac{1}{2}$

b) $\csc\theta = -1$ d) $\tan\theta = -\frac{\sqrt{3}}{3}$ f) $\sec\theta = 0$

2. Solve the following trig equations by collecting like terms:

a) $\sin\theta = -\sin\theta + \sqrt{3}$ b) $2\cos\theta - 1 = \cos\theta - 2$

3. Use a substitution (let $a = \text{trig}\theta$) to solve the resulting quadratic equation for a; then solve for θ from the resulting one-step equation(s).

a) $\sin^2\theta = 1$ c) $3\tan^2\theta - 1 = 0$ e) $\sec^4\theta - 4\sec^2\theta = 0$

b) $2\cos^2\theta - 1 = 0$ d) $2\sin^2\theta + 3\sin\theta + 1 = 0$ f) $\sqrt{3}\cot^2\theta = -\cot\theta$

4. Factor the common term first and then solve for θ:

a) $\sin\theta\cos\theta + \cos\theta = 0$ c) $\cot\theta\csc\theta + 2\cot\theta = 0$

b) $\csc\theta\cos\theta + \cos\theta = 0$ d) $\sqrt{3}\sin\theta\cos\theta - 2\sin\theta = 0$

5. Use a substitution for the given angle (i.e., let $\alpha = 2\theta$) and solve the resulting trig equation (in terms of α) for all coterminal angles to α. Then, back substitute in order to solve for θ both for one revolution and all coterminal angles to θ.

a) $\sin\left(\theta - \frac{\pi}{4}\right) = -1$ c) $\sin 2\theta = 1$ e) $\tan\frac{\theta}{2} = 1$

b) $2\cos\left(\theta + \frac{2\pi}{3}\right) + 1 = 0$ d) $\cos^2 3\theta = 1$ f) $\cot\frac{2\theta}{3} = -\sqrt{3}$

6. Rewrite the entire equation in terms of only one trig function using various identities (reciprocal, quotient, cofunction, Pythagorean or negative angle). Solve the remaining equation for θ.

a) $2\tan\theta = \frac{1}{\cot\theta} + \frac{\sqrt{3}}{3}$

b) $\sin(-\theta) + \tan(-\theta)\cos(-\theta) - \sqrt{2} = 0$

c) $\sin\theta + \cos\left(\frac{\pi}{2} - \theta\right) + 1 = 0$

d) $2\sin^2\theta - \cos\theta - 1 = 0$

7. Square both sides of the equation (properly!) in order to obtain squared trig functions, one of which can be replaced in terms of the other via the Pythagorean identities. Solve the resulting quadratic equation for θ.

a) $\cos\theta + 1 = \sin\theta$

b) $\tan\theta - \sec\theta = 1$

NOTE: Squaring both sides of the equation may introduce extraneous (invalid) solutions. Check all possible answers in the ORIGINAL equation in order to determine the valid solutions to the original equation.

PART B – Proofs of trigonometric identities. Use PROPER LS = RS FORMAT. Do not hand these questions in without checking your proofs with the teacher.

8. $[1 + \sin x][1 + \sin(-x)] = \cos^2 x$

9. $\sin x \tan x = \sec x - \cos x$

10. $\sin^2 x + \sin^2\left(\frac{\pi}{2} - x\right) = 1$

11. $\csc(A + B)\cos\left(\frac{\pi}{2} - A - B\right) = 1$

12. $\csc^4 x - \cot^4 x = 2\csc^2 x - 1$

13. $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

PART C – Sketch the following graphs on trig graph paper. State: (a) all transformations, (b) period and amplitude and (c) locations of all zeros, max/min points and VA.

14. $y = \sin\left(-2x + \frac{\pi}{2}\right)$

15. $y = 2\sec x + 1$

16. $y = \tan\left(\frac{\pi}{2} - x\right)$