Precalculus: Unit 5 Test   Name: ___________________

PART A – No Calculator Multiple Choice: Write answers on the answer sheet.

1. If f(x) = sin(x) and if g(x) = cos(x), then f(2x) =

   a. 2 f(x)
   b. f(2) g(x)
   c. f(x) g(x)
   d. 2 f(x) g(x)
   e. f(2) g(x) + g(2) f(x)

2. Evaluate sin(22.5°)

   a. $\frac{\sqrt{2}}{4}$
   b. $\frac{\sqrt{3}}{4}$
   c. $\frac{\sqrt{6}-\sqrt{2}}{4}$
   d. $\sqrt{\frac{2-\sqrt{2}}{2}}$
   e. $\frac{\sqrt{2-\sqrt{2}}}{2}$

3. How many numbers between 0 and 2π satisfy the equation sin(2x) = cos(x)?

   a. None
   b. One
   c. Two
   d. Three
   e. Four

4. If $\cos A \cos B = \sin A \sin B$, then $\cos(A+B)$

   a. 0
   b. 1
   c. 2
   d. $\cos A + \cos B$
   e. $\cos A \cos B + \sin A \sin B$

5. The function f(x) = sin(x) cos(2x) + cos(x) sin(2x) has an amplitude of

   a. 1
   b. 1.5
   c. 2
   d. 3
   e. 6

6. Determine the period of the same function f(x) = sin(x) cos(2x) + cos(x) sin(2x), if possible:

   a. π/3
   b. π/2
   c. 2π/3
   d. 2π
   e. this function is not sinusoidal and therefore is NOT periodic.

7. A function with the property $f(1+2)=\dfrac{f(1)+f(2)}{1-f(1)f(2)}$ is:

   a. f(x) = sin(x)
   b. f(x) = tan(x)
   c. f(x) = sec(x)
   d. f(x) = e^x
   e. f(x) = -1

8. Which of the following could not be set equal to sin(x) as an identity?

   a. $\cos\left(\dfrac{\pi}{2}-x\right)$
   b. $\cos\left(x-\dfrac{\pi}{2}\right)$
   c. $\sqrt{1-\cos^2 x}$
   d. tan(x) sec(x)
   e. –sin(-x)

9. A simpler equation for $(\sec x-1)(\sec x+1)$ is

   a. $\sin^2(x)$
   b. $\cos^2(x)$
   c. $\tan^2(x)$
   d. $\cot^2(x)$
   e. $\sec^2(x)$

10. How many numbers between 0 and 2π solve the equation $3\cos^2(x) + \cos(x) = 2$?

   a. None
   b. One
   c. Two
   d. Three
   e. Four

11. If $f(x) = g(x)$ is an identity and $\dfrac{f(x)}{g(x)}=k$, which of the following must be false?

   a. $g(x) \neq 0$
   b. $f(x) = 0$
   c. $k = 1$
   d. $f(x) - g(x) = 0$
   e. $f(x)\, g(x) > 0$

12. The equation that produces the solutions $\theta = 150^o, 330^o$ is

   A) $\sin\theta=\dfrac{1}{2}$

   B) $\sec\theta=\dfrac{2\sqrt{3}}{3}$

   C) $\tan\theta=-\dfrac{\sqrt{3}}{3}$

   D) $\cos\theta=-\dfrac{\sqrt{3}}{2}$

PART B – No Calculator Free Response: Write answers on a separate sheet of paper.

1. Evaluate $\cos 165^{o}$ exactly.

2. Evaluate the following exactly.

   a) $\dfrac{\cot^2 x - \csc^2 x}{2}$, for any $x \in$ domain of validity

   b) $\left(1-2\sin^2 x\right)^2 + 4\sin^2 x\cos^2 x$

   c) $\sin\left(\dfrac{\pi}{2}-\theta\right)$, given that $\sec\theta = \dfrac{7}{4}$

   d) $\sin 50^{o} \cdot \sec 40^{o}$

3. Evaluate $\sin\left(\cos^{-1}\dfrac{3}{5} + \sin^{-1}\dfrac{3}{5}\right)$ exactly.

4. Find the exact solution for $\sin^2\left(\dfrac{x}{2}\right) = \cos 2x$ on $[0, 2\pi]$.

5. Given the trigonometric equation sin(3x) = 3sin(x)cos(x), show numerically whether or not the equation represents an identity.

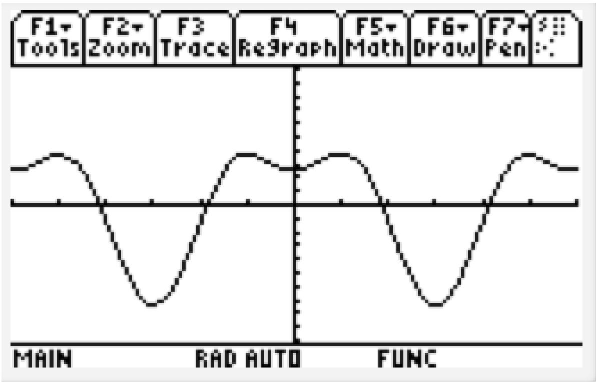
6. Prove the following identities.

   a) $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

   b) $2\csc 2x = \csc^2 x \tan x$

   c) $\dfrac{\tan x + \sin x}{2\tan x} = \cos^2\left(\dfrac{x}{2}\right)$

7. The graph of $f(x) = \cos x - \frac{1}{2}\cos 2x$ is shown in the diagram below on the window



The x-values that correspond to the local maximum and minimum points of f(x) are zeroes of a second equation g(x) = sin(x) – sin(2x).

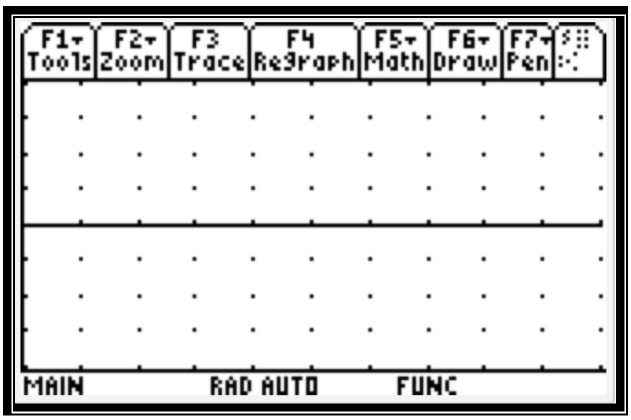
(a) On the diagram of $f(x) = \cos x - \frac{1}{2}\cos 2x$ above, put a box around any local maximum point and a circle around any local minimum point.

(b) Determine the zeroes of g(x) and determine the x co-ordinate of your selected maximum and minimum points.

PART C – Calculator Active Free Response: Answer in the space provided.

8. Given the following trigonometric equation, sec(x) – sin(x) tan(x) = cos(x), show whether or not the equation represents an identity by the following methods:

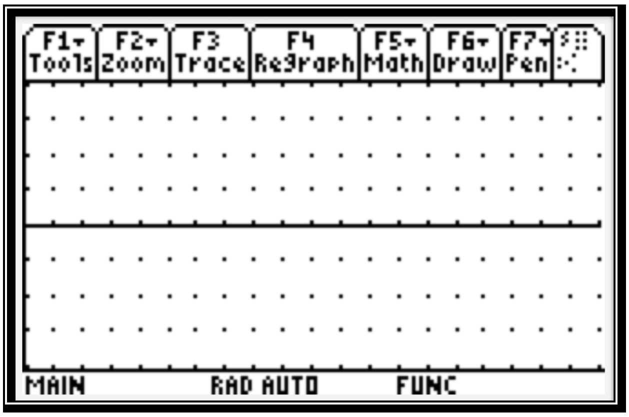
A) Graphically (Show graph and include a description of how you used the GDC to determine your answer)
B) Algebraically



9. Solve by factoring the equation $4\tan^2\theta + \frac{4}{\cot\theta} + \sin\theta\csc\theta = 0$ on [0,2π]. State the exact solutions and approximate to three decimal places. Show all steps of your solution. State the syntax for solving this equation from the home screen of the TI-89 on the domain of [0,2π].

10. The function $f(x) = 4\sin x\cos^3 x - 4\sin^3 x\cos x$ is given.

a) Sketch $f(x)$ in the above window with $x \in [0, 2\pi]$, $y \in [-2, 2]$, and $x_{scale} = \frac{\pi}{4}$.

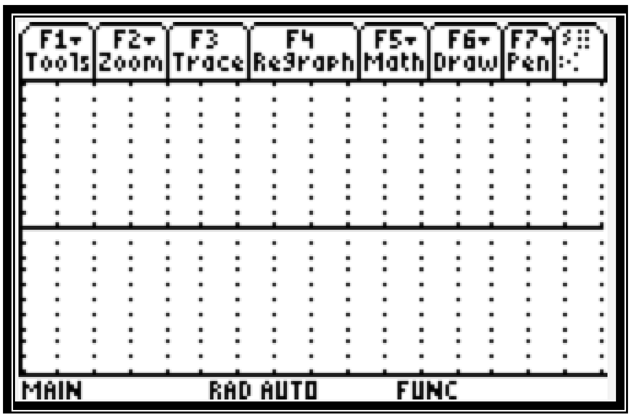


b) Write the equation of the sinusoid f(x) in terms of sine or cosine. Briefly, in one sentence, explain how you determined the sine/cosine equation.

c) State an identity for $f(x) = 4\sin x\cos^3 x - 4\sin^3 x\cos x$ and prove it algebraically.

d) Approximate the solution to the equation $\sqrt{5}\cdot f(x) - 2 = 0$ on the specified domain of $[0,\pi]$ to three decimal places. Explain in one sentence how you used your calculator to solve this equation.

11. Solve $\cos^3(x) - 2\sin(x) = \frac{1}{\sqrt{2}}$ on $[0,2\pi]$. Briefly explain how you solved the equation. Include diagrams, commands, displays etc… in your explanation. A grid is included for any diagrams in your solution.



<u>BONUS</u>: Show work/answer on the answer sheet.

Solve $2^{2(\cos 2x)+1} - 5\cdot 2^{\cos 2x} = -2$ for $x \in$ ° .