25 The mass m kilograms of a radioactive substance at time t days is given by $m = 5e^{-0.13t}$.

a) What is the initial mass?

b) How long does it take for the substance to decay to 0.5 kg? Give the answer in days accurate to 3 significant figures.

In questions 26–36, solve for x in the logarithmic equation. Give exact answers and be sure to check for extraneous solutions.

26 $\log_2(3x - 4) = 4$

27 $\log(x - 4) = 2$

28 $\ln x = -3$

29 $\log_{16} x = \frac{1}{2}$

30 $\log \sqrt{x + 2} = 1$

31 $\ln(x^2) = 16$

32 $\log_2(x^2 + 8) = \log_2 x + \log_2 6$

33 $\log_3(x - 8) + \log_3 x = 2$

34 $\log 7 - \log(4x + 5) + \log(2x - 3) = 0$

35 $\log_3 x + \log_3(x - 2) = 1$

36 $\log x^8 = (\log x)^4$

In questions 37–40, solve each inequality.

37 $5 \log x + 2 > 0$

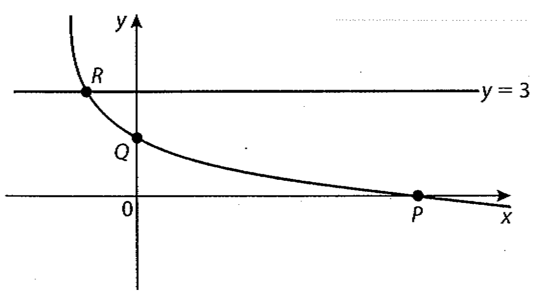
38 $2\log x^2 - 3\log x < \log 8x - \log 4x$

39 $(e^x - 2)(e^x - 3) < 2e^x$

40 $3 + \ln x > e^x$

Practice questions

1 A portion of the graph $y = 2 - \log_3(x + 1)$ is shown. It intersects the x-axis at point P, the y-axis at point Q and the line $y = 3$ at point R. Find the following:



a) The x-coordinate of point P.

b) The y-coordinate of point Q.

c) The coordinates of point R.

2 The amount $A(t)$, in grams, of a certain radioactive substance remaining after t years decays by the formula $A(t) = A_0e^{-0.0045t}$, where A_0 is the initial amount.

a) If 5 grams are left after 800 years, how many grams were present initially?

b) What is the half-life of the substance?

3 a) Find expressions for the nth term and the sum to n terms of the following arithmetic series, $\ln y + \ln y^2 + \ln y^3 + \ldots$ where $y > 0$.

b) Hence, find expressions for the nth term and the sum to n terms of the following arithmetic series, $\ln(xy) + \ln(xy^2) + \ln(xy^3) + \ldots$ where $x > 0$ and $y > 0$.

4 Solve, for x, the equation $\log_2(5x^2 - x - 2) = 2 + 2\log_2 x$.

5 If $\log_2 4\sqrt{2} = x$, $\log_z y = 4$, and $y = 4x^2 - 2x - 6 + z$, find y.

6 Find the **exact** values of t for which $2e^{3t} - 7e^{2t} + 7e^t = 2$.

7 Find the **exact** solution(s) to the equation $8e^2 - 2e\ln x = (\ln x)^2$.

8 Find the exact value of x for each equation.

a) $\log_3 x - 4\log_x 3 + 3 = 0$

b) $\log_2(x - 5) + \log_2(x + 2) = 3$

9 Express each as a single logarithm.

a) $2\log a + 3\log b - \log c$

b) $3\ln x - \frac{1}{2}\ln y + 1$

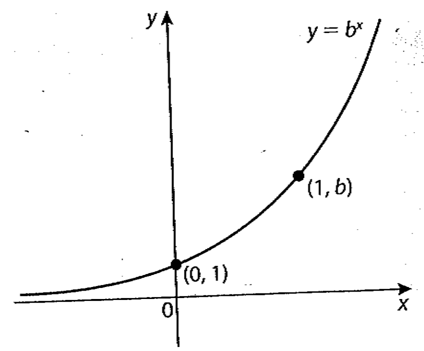
10 A piece of wood is recovered from an ancient building during an archaeological excavation. The formula $A(t) = A_0e^{-0.000\,124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years. For the ancient piece of wood it is found that $A(t)$ is 79% of the amount of the carbon in a living tree. How old is the piece of wood, to the nearest 100 years?

11 The graph of the equation $y = \log_3(2x - 3) - 4$ intersects the x-axis at the point $(c, 0)$. Without using your GDC, find the exact value of c.

12 The graph of $y = b^x$, $b > 1$ is shown. On separate coordinate planes, sketch the graphs of

a) $y = b^{-x}$

b) $y = b^{1-x}$



13 Radium decays exponentially and its half-life is 1600 years. If A_0 represents the initial amount of radium in a sample and $A(t)$ represents the amount remaining after t years, then $A(t) = A_0e^{-kt}$.

a) Find the value of k approximated to four significant figures.

b) Find what percentage of the original amount of radium will be remaining after 4000 years.

14 Solve the equation $e^{-x} - x + 1 = 0$.

15 Find the set of values of x for which $|0.1x^2 - 2x + 3| < \log_{10} x$.

16 Determine the values of x that satisfy the inequality $\dfrac{xe^x}{x^2 - 1} \geqslant 1$.

17 a) Solve the equation $2(4^x) + 4^{-x} = 3$.

b) (i) Solve the equation $a^x = e^{2x+1}$ where $a > 0$, giving your answer for x in terms of a.

(ii) For what value of a does the equation have no solution?

18 The solution of $2^{2x+3} = 2^{x+1} + 3$ can be expressed in the form $a + \log_2 b$ where $a, b \in \mathbb{Z}$. Find the value of a and of b.

19 Solve $2(\ln x)^2 = 3\ln x - 1$ for x. Give your answers in **exact** form.

20 A sum of \$100 is invested.

a) If the interest is compounded annually at a rate of 5% per year, find the total value V of the investment after 20 years.

b) If the interest is compounded monthly at a rate of $\frac{5}{12}$% per month, find the minimum number of months for the value of the investment to exceed V.

21 Solve the equation $9\log_5 x = 25\log_x 5$ expressing your answer in the form $5^{\frac{p}{q}}$, where $p, q \in \mathbb{Z}$.

22 Solve $|\ln(x + 3)| = 1$. Give your answers in **exact** form.

23 Solve the equation $\left|e^{2x} - \frac{1}{x+2}\right| = 2$.

24 An experiment is carried out in which the number n of bacteria in a liquid, is given by the formula $n = 650e^{kt}$, where t is the time in minutes after the beginning of the experiment and k is a constant. The number of bacteria doubles every 20 minutes. Find the exact value of k.

25 The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x - 2) - \ln(x^2 - 4)$.

a) Express $f(x)$ in the form $\ln\left(\frac{x}{x+a}\right)$.

b) Find an expression for $f^{-1}(x)$.

26 a) The function f is defined by $f: x \mapsto e^x - 1 - x$.

(i) Use your GDC to find the minimum value of f.

(ii) Prove that $e^x \geqslant 1 + x$ for all real values of x.

b) Use mathematical induction to prove that

$$(1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{n}\right) = n + 1 \text{ for all integers } n \geqslant 1$$

c) Use the results of parts a) and b) to prove that

$$e^{\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right)} > n$$

d) Find a value of n for which

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} > 100$$

3 a) Nick: 20 Charlotte: 17.6
b) Nick: 390 Charlotte: 381.3
c) Charlotte will exceed the 40 hours during week 14.
d) In week 12 Charlotte will catch up with Nick and exceed him.

4 a) Loss for the second month = 1060 g
Loss for the third month = 1123.6 g
b) Plan A loss = 1880 g
Plan B loss = 1898.3 g
c) (i) Loss due to plan A in all 12 months = 17 280 g
(ii) Loss due to Plan B in all 12 months = 16 869.9 g

5 a) €895.42 b) €6985.82

6 a) 142.5 b) 19 003.5

7 $1, \sqrt[3]{7}, 1, 1, \sqrt[3]{7}, 1, \ldots; 2, 0, 2, 0, 2, \ldots$

8 a) On the 37th day b) 407 km

9 a) 1.5 b) 207 595
c) 2009 d) 619 583
e) Market saturation

10 −4, 3006

11 a) $\sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$ b) $\frac{1}{2}$
c) (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ d) (i) $\frac{1}{512}$ (ii) 2

12 a) 1220 b) 36 920

13 a) Area A = 1, Area B = $\frac{1}{9}$ b) $\frac{1}{81}$
c) $1 + \frac{8}{9}, 1 + \frac{8}{9} + \left(\frac{8}{9}\right)^2$ d) 0

14 a) Neither, geometric converging, arithmetic, geometric diverging
b) 6

15 a) (i) Kell: 18 400, 18 800; YBO: 18 190, 19 463.3
(ii) Kell: 198 000; YBO: 234 879.62
(iii) Kell: 21 600; YBO: 31 253.81
b) (i) After the second year
(ii) 4th year

16 a) 62 b) 936

17 a) $7000(1 + 0.0525)^t$ b) 7 years
c) Yes, since 10 084.7 > 10 015.0

18 a) 11 b) 2 c) 15

19 15, −8 20 −2, −7 21 10 300

22 Proof

23 a) $a_n = 8n - 3$ b) 50

24 2 099 520

25 $6n - 5$ 26 72 27 559

28 −3, 3 29 9 30 62

31 $-\frac{36}{5}$

32 a) 4 b) $16(4^n - 1)$

33 a) $|x| < 1.5$ b) 5

34 3168

35 a) $\frac{n(3n+1)}{2}$ b) 30

36 −7

37 1275 ln 2

38 a) 4, 8, 16
b) (i) $u_n = 2^n$ (ii) proof

39 a) $\frac{2}{3}$ b) 9

40 2, −3 41 55 42 −2, 4

43 $\frac{\theta}{1 - \cos\theta}$

44 a) 1, 5, 9 b) $4n - 3$

45 a) $32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$
a) 32.808 040 1001

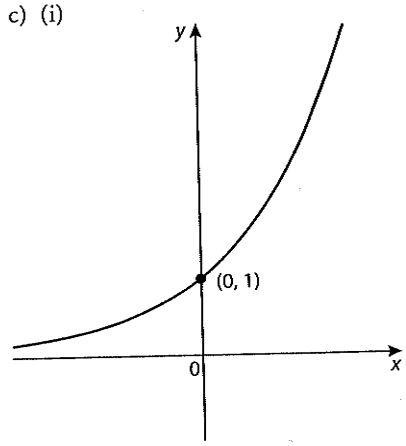
46 a) $5000(1.063)^n$ b) 6786.35
c) (i) $5000(1.063)^n > 1000$ (ii) 12

47 Proof 48 7

Chapter 5

Exercise 5.1 and 5.2

1 a) $y = b^x$
b) Domain $\{x : x \in \mathbb{R}\}$, range $\{y : y > 0\}$
c) (i)



(ii)